SENSITIVITY ANALYSIS OF NONLINEAR CIRCUITS

by

Ronald Alvin Lee

United States Naval Postgraduate School

THESIS

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Ronald Alvin Lee
Lieutenant, United States Navy
B.A., Duke University, 1962

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ABSTRACT

The use of the digital computer for the simultaneous generation of circuit responses and their corresponding sensitivity to parameter changes is considered for circuits containing diodes and transistors. Modeling of the diodes and transistors is based upon the Ebers-Moll equations, with the addition of voltage-dependent capacitors to account for switching time. The resulting general iterative equations are then used to calculate the effects of radiation on a p-n junction. The waveforms and circuit recovery time are studied as a function of circuit parameters. The use of sensitivity functions for predicting waveform variations is considered.

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I. INTRODUCTION

A measure of the sensitivity of circuit-response variables to parameter variations is useful in circuit analysis and design. Many different definitions of sensitivity are available. H. Bode [1] defines a logarithmic or normalized sensitivity function of a variable, $x_i(t,a_j)$, with respect to a parameter, a_j, as $S_{a_j}^{x_i} = \frac{\partial \ln a_j}{\partial \ln x_i}$. The inverse of this definition is adopted by I. M. Horowitz [2]. Calculation of the sensitivity of linear-circuit responses to incremental changes in network parameters has been presented by J. Leeds [3]. Recently S. R. Parker [4] generalized the technique to include nonlinear circuits. Both Leeds and Parker define the sensitivity function as $S_{a_j}^{x_i} = \frac{\partial x_i}{\partial a_j}$. This work was preceded by that of P. Kokotović [5] and R. Tomović [6] who considered variations in the solution of a linear differential equation due to incremental changes in the coefficients. Tomović defines the sensitivity coefficients as $S_{a_j}^{x_i} = \frac{\partial x_i}{\partial a_j}$, and Kokotović adopts the definition $S_{a_j}^{x_i} = \frac{\partial x_i}{\partial \ln a_j}$, and discusses implementation of the sensitivity coefficient using analog computation. The work by Leeds and Parker is oriented towards the digital computer. In this thesis digital calculation of sensitivity functions of nonlinear circuits is considered in detail with specific application to the study of the effects of radiation on solid-state junctions. The sensitivity function is defined as $S_{a_j}^{x_i} = \frac{\partial x_i}{\partial \ln a_j} = \frac{\partial x_i}{(\partial a_j/a_j)}$. This definition is adopted since it enables sensitivity functions with respect to parameters having widely different nominal values to be conveniently compared on a percentage basis, and also avoids some difficulties encountered in the definitions adopted by Bode and Horowitz when the circuit-response variable is zero. It should be noted that the above definitions of the sensitivity function apply only for infinitesimally small changes in x_i and a_j. The practicality of this definition when applied to specific examples with large parameter increments is considered in this thesis.

In Chapter II it is shown that for linear circuits the sensitivity function may be derived using the compensation theorem [7]. A convenient set of iterative equations is developed using the concept of state variables which allows the time response of the states and their corresponding sensitivities to be obtained simultaneously. A linear example illustrates the procedure.

In order to obtain useful results from the sophisticated circuit analysis and design programs for nonlinear circuits, acceptable nonlinear models are required. Chapter III discusses modeling of diodes and transistors for digital computer analysis based on the Ebers-Moll [8] equations. These models contain nonlinear capacitances and nonlinear resistances for the p-n junction. A state-variable formulation is developed which results in a set of iterative equations whose solution yields time responses for circuits containing linear elements, diodes, and transistors. The procedure is illustrated by considering the effect of a radiation current pulse injected across (1) a p-n junction diode, and (2) the base-emitter p-n junction of a transistor operating in the active region. These examples yield new results concerning the effect of radiation on p-n junctions and comparative data for several runs is presented.

Sensitivity analysis of nonlinear circuits is discussed in Chapter IV. The state-variable formulation developed in Chapter III is extended, resulting in a set of iterative equations which allow time responses and their corresponding sensitivities to be obtained simultaneously. These procedures are applied to the diode example of the previous chapter and the results interpreted in terms of the sensitivity of the diode response to percentage parameter variations.

II. SENSITIVITY ANALYSIS OF LINEAR CIRCUITS

A. INTRODUCTION

There are several general theorems which are useful in analysis and design of linear networks. One of these, the compensation theorem [7], has been known for many years and may be applied when it is desired to determine what effect an incremental impedance change in one branch of a network has upon the currents and voltages of other branches of the network. Recently the concept of the sensitivity of certain circuit-response variables to incremental changes in given network parameters has been presented by J. Leeds [3]. This work was preceded by that of P. Kokotović [5] and R. Tomović [6] who considered variations in the solution of a linear differential equation due to incremental changes in the coefficients. Tomović and Kokotović implemented the sensitivity coefficient by using analog computation. Leeds' work is oriented towards the digital computer.

In the next section sensitivity analysis of linear circuits is considered. In the succeeding section the relationship between the older compensation theorem and the recent sensitivity concept is discussed. This correlation has not appeared in the literature heretofore. The final section contains an illustrative example to demonstrate the implementation of the results on a digital computer.

B. DETERMINATION OF SENSITIVITY FUNCTIONS

1. Auxiliary Network Approach

Leeds [3] developed a relationship between the sensitivity function, $S_{a_j}^{x_i} = \frac{\partial x_i}{\partial a_j}$ (where x_i is a response variable and a_j is a circuit parameter), and node voltages and branch currents of an auxiliary network called the sensitivity model. The auxiliary network is obtained by reducing all independent sources to zero and adding a single source in the branch containing the element under consideration. The source depends upon the nature of the element. Fig. 2.1 summarizes the sources to be added for branches containing resistance, inductance, and capacitance respectively. The voltages and currents in the auxiliary network are the voltage and current sensitivities with respect to the parameter being considered.

The advantage of Leeds' approach is that it allows the sensitivity of a network to be calculated by evaluating the responses of a coupled auxiliary network rather than actually performing the differentiation indicated by the definition of the sensitivity function.

2. Augmented State Equations

Sensitivity functions may also be obtained using the concepts of state variables. The network is first represented in state-variable form as discussed by T. Bashkow [9], and Kuh and Rohrer [10].

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \tag{2.1}$$

where

$\underline{x}$ is an n vector of circuit states

A is a constant nxn matrix

$\underline{u}$ is an m vector of forcing functions.

The sensitivity functions, $S_{a_j}^{x_i} = \frac{\partial x_i}{\partial \ln a_j}$, may be obtained by performing the indicated partial differentiation on (2.1). The resulting partial derivatives can be considered as additional states which may be solved in conjunction with the original state equations. They may be included in the following augmented matrix;

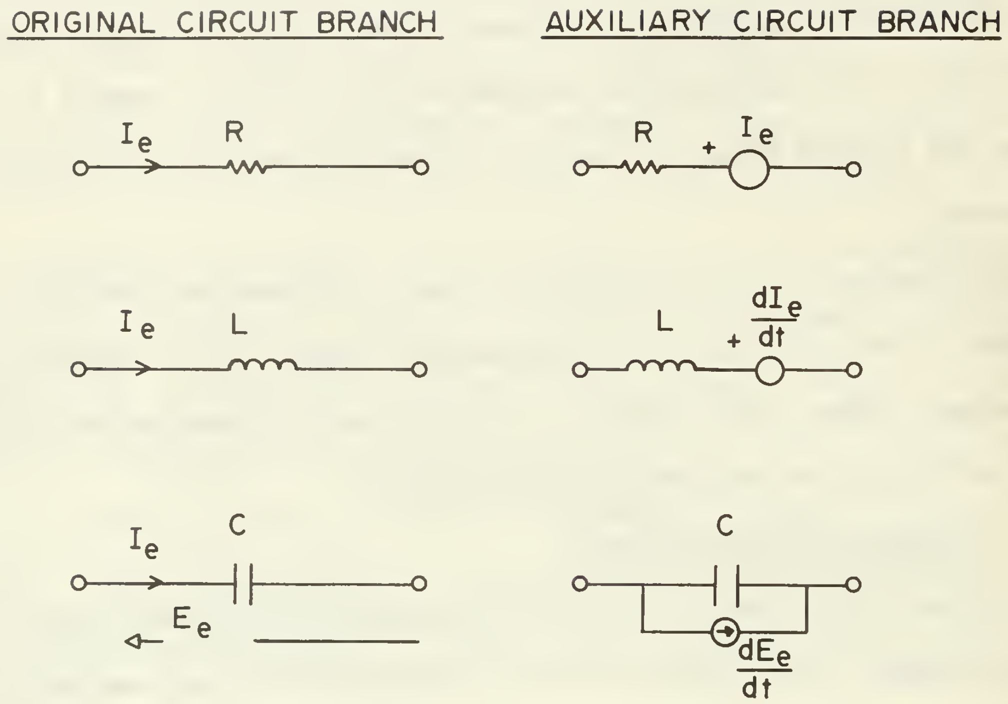


FIG. 2.1. CIRCUIT PARAMETER BRANCHES AND SENSITIVITY MODEL BRANCHES.

$$\begin{bmatrix} \dot{\underline{x}} \\ \dot{\underline{x}}_{s1} \\ \vdots \\ \dot{\underline{x}}_{sj} \\ \vdots \\ \dot{\underline{x}}_{sr} \end{bmatrix} = \begin{bmatrix} A & & & & 0 \\ & \ddots & & & \\ & & A & & \\ & & & \ddots & \\ 0 & & & & A \end{bmatrix} \begin{bmatrix} \underline{x} \\ \underline{x}_{s1} \\ \vdots \\ \underline{x}_{sj} \\ \vdots \\ \underline{x}_{sr} \end{bmatrix} + \begin{bmatrix} B & & & & 0 \\ & \ddots & & & \\ & & B & & \\ & & & \ddots & \\ 0 & & & & B \end{bmatrix} \begin{bmatrix} \underline{u} \\ \underline{u}_{s1} \\ \vdots \\ \underline{u}_{sj} \\ \vdots \\ \underline{u}_{sr} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ A_{s1} & B_{s1} \\ \vdots & \vdots \\ A_{sj} & B_{sj} \\ \vdots & \vdots \\ A_{sr} & B_{sr} \end{bmatrix} \begin{bmatrix} \underline{x} \\ \underline{u} \end{bmatrix} \quad (2.2)$$

or in partitioned form

$$\begin{bmatrix} \dot{\underline{x}} \\ \hline \dot{\underline{x}}_{sj} \end{bmatrix} = \left[\begin{array}{c|c} A & 0 \\ \hline 0 & A_D \end{array}\right] \begin{bmatrix} \underline{x} \\ \hline \underline{x}_{sj} \end{bmatrix} + \left[\begin{array}{c|c} B & 0 \\ \hline 0 & B_D \end{array}\right] \begin{bmatrix} \underline{u} \\ \hline \underline{u}_{sj} \end{bmatrix} + \left[\begin{array}{c|c} 0 & 0 \\ \hline A_{sj} & B_{sj} \end{array}\right] \begin{bmatrix} \underline{x} \\ \hline \underline{u} \end{bmatrix} \quad (2.3a)$$

where $j = 1,2,\ldots,r$, and r is the number of sensitivity parameters and

$$\underline{x}_{sj} = \begin{bmatrix} \frac{\partial x_1}{\partial \ln a_j} & \cdots & \frac{\partial x_n}{\partial \ln a_j} \end{bmatrix}^T \quad (2.3b)$$

$$\underline{u}_{sj} = \begin{bmatrix} \frac{\partial u_1}{\partial \ln a_j} & \cdots & \frac{\partial u_m}{\partial \ln a_j} \end{bmatrix}^T \quad (2.3c)$$

$$A_{sj} = \frac{\partial A}{\partial \ln a_j} = \begin{bmatrix} \frac{\partial a_{11}}{\partial \ln a_j} & \cdots & \frac{\partial a_{1n}}{\partial \ln a_j} \\ \vdots & & \vdots \\ \frac{\partial a_{n1}}{\partial \ln a_j} & \cdots & \frac{\partial a_{nn}}{\partial \ln a_j} \end{bmatrix} \quad (2.3d)$$

$$B_{sj} = \frac{\partial B}{\partial \ln a_j} = \begin{bmatrix} \frac{\partial b_{11}}{\partial \ln a_j} & \cdots & \frac{\partial b_{1m}}{\partial \ln a_j} \\ \vdots & & \vdots \\ \frac{\partial b_{n1}}{\partial \ln a_j} & \cdots & \frac{\partial b_{nm}}{\partial \ln a_j} \end{bmatrix} \quad (2.3e)$$

$$A_D = \begin{bmatrix} A & & & & \\ & \ddots & & 0 & \\ & & A & & \\ & 0 & & \ddots & \\ & & & & A \end{bmatrix} \tag{2.3f}$$

$$B_D = \begin{bmatrix} B & & & & \\ & \ddots & & 0 & \\ & & B & & \\ & 0 & & \ddots & \\ & & & & B \end{bmatrix} \tag{2.3g}$$

If $\underline{u}$ is independent of a_j, that is if a_j is a circuit parameter only, (2.3a) takes the form

$$\begin{bmatrix} \dot{\underline{x}} \\ \hline \dot{\underline{x}}_{sj} \end{bmatrix} = \left[\begin{array}{c|c} A & 0 \\ 0 & A_D \end{array}\right] \begin{bmatrix} \underline{x} \\ \hline \underline{x}_{sj} \end{bmatrix} + \left[\begin{array}{c|c} 0 & B \\ \hline A_{sj} & B_{sj} \end{array}\right] \begin{bmatrix} \underline{x} \\ \hline \underline{u} \end{bmatrix} \tag{2.4}$$

Separating the set of first-order partial differential equations of (2.3a) for each sensitivity parameter yields

$$\dot{\underline{x}}_{sj} = A\,\underline{x}_{sj} + B\,\underline{u}_{sj} + A_{sj}\,\underline{x} + B_{sj}\,\underline{u} \tag{2.5}$$

$$\text{for } j = 1,2,\ \ldots,\ r$$

Trapezoidal integration is commonly used to solve a set of first-order matrix differential equations because the procedure always converges and provides acceptable accuracy for most problems [11]. Utilizing trapezoidal integration, (2.1) and (2.5) yield the following iterative solutions:

$$\underline{x}(nT) = \phi(T)\underline{x}((n-1)T) + \Gamma(T)[\underline{u}(nT)+\underline{u}((n-1)T)] \tag{2.6}$$

$$\begin{aligned} \underline{x}_{sj}(nT) = {} & \phi(T)\underline{x}_{sj}((n-1)T) + \Gamma(T)[\underline{u}_{sj}(nT)+\underline{u}_{sj}((n-1)T)] \\ & + \Gamma_{1j}(T)[\underline{x}(nT)+\underline{x}((n-1)T)] + \Gamma_{2j}(T)[\underline{u}(nT)+\underline{u}((n-1)T)] \\ & \text{for } j=1,2,\ \ldots\ ,r \end{aligned} \tag{2.7}$$

where

$$\phi(T) = [I - \frac{T}{2} A]^{-1}[I + \frac{T}{2} A]$$

$$\Gamma(T) = [I - \frac{T}{2} A]^{-1} \frac{BT}{2}$$

$$\Gamma_{1j}(T) = [I - \frac{T}{2} A]^{-1} \frac{A_{sj}\ T}{2}$$

$$\Gamma_{2j}(T) = [I - \frac{T}{2} A]^{-1} \frac{B_{sj}\ T}{2}$$

Equations (2.6) and (2.7) are a set of iterative equations whose solution yields the time response of the states and their corresponding sensitivities. Combining the results into a single equation results in a solution format similar to (2.3).

$$\begin{bmatrix} \underline{x}(nT) \\ \hline \underline{x}_{sj}(nT) \end{bmatrix} = \left[\begin{array}{c|c} \phi(T) & 0 \\ \hline 0 & \phi_D(T) \end{array}\right] \begin{bmatrix} \underline{x}[(n-1)T] \\ \hline \underline{x}_{sj}[(n-1)T] \end{bmatrix} + \left[\begin{array}{c|c} \Gamma(T) & 0 \\ \hline 0 & \Gamma_D(T) \end{array}\right] \begin{bmatrix} (\underline{u}(nT) + \underline{u}[(n-1)T]) \\ \hline (\underline{u}_{sj}(nT) + u_{sj}[(n-1)T]) \end{bmatrix}$$

$$\left[\begin{array}{c|c} 0 & 0 \\ \hline \Gamma_{1j}(T) & \Gamma_{2j}(T) \end{array}\right] \begin{bmatrix} (\underline{x}(nT) + \underline{x}[(n-1)T]) \\ \hline (\underline{x}_{sj}(nT) + \underline{x}_{sj}[(n-1)T]) \end{bmatrix} \qquad (2.8a)$$

for $j = 1,2, \ldots, r$

where

$$\phi_D(T) = \begin{bmatrix} \phi(T) & & & 0 \\ & \ddots & & \\ & & \phi(T) & \\ 0 & & & \ddots \\ & & & & \phi(T) \end{bmatrix} \qquad (2.8b)$$

$$\Gamma_D(T) = \begin{bmatrix} \Gamma(T) & & & 0 \\ & \ddots & & \\ & & \Gamma(T) & \\ 0 & & & \ddots \\ & & & & \Gamma(T) \end{bmatrix} \qquad (2.8c)$$

C. USE OF THE COMPENSATION THEOREM TO DERIVE SENSITIVITY FUNCTIONS

The compensation theorem relates an incremental impedance change in one branch of a network with corresponding incremental current and voltage changes in the other branches [7]. In order to relate the compensation theorem directly with sensitivity analysis an analagous relation for incremental changes in the branch parameters R, L, and C is developed.

1. Resistive Change

If the resistance in a branch is changed by an amount ΔR, and a voltage source equal to $I\cdot(\Delta R)$ is introduced (Fig. 2.2a) the network variables remain unchanged. If another source is introduced (Fig. 2.2b) the branch voltage and current will change by an incremental amount ΔV and ΔI respectively. By applying the superposition theorem (Fig. 2.2c) the incremental voltage and current can be calculated.

$$\Delta V = \Delta I\cdot(R + \Delta R) + I\cdot(\Delta R) \tag{2.9}$$

Dividing (2.9) by ΔR and letting $\Delta R \to 0$ results in (2.10).

$$\frac{\partial V}{\partial R} = \frac{\partial I}{\partial R} R + I \tag{2.10}$$

2. Inductive Change

If the inductance in a branch is changed by an amount ΔL, and a voltage source equal to $\Delta L \cdot (\frac{dI}{dt})$ is introduced (Fig. 2.3a) the network variables remain unchanged. If another source is introduced (Fig. 2.3b) the branch voltage and current will change by an incremental amount ΔV and ΔI respectively. By applying the superposition theorem (Fig. 2.3c) the incremental current and voltage can be calculated.

$$\Delta V = (L + \Delta L)\frac{d\Delta I}{dt} + \Delta L\frac{dI}{dt} \tag{2.11}$$

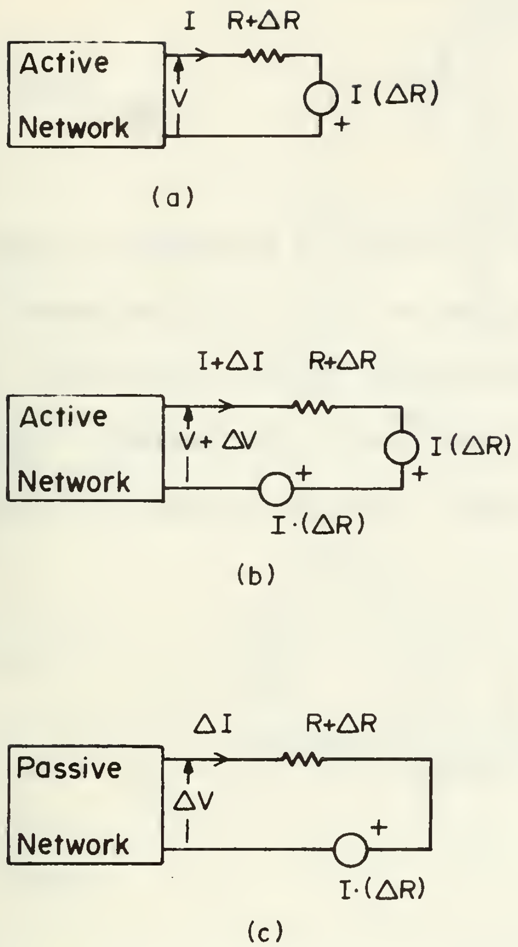


FIG. 2.2 RESISTIVE CHANGES.

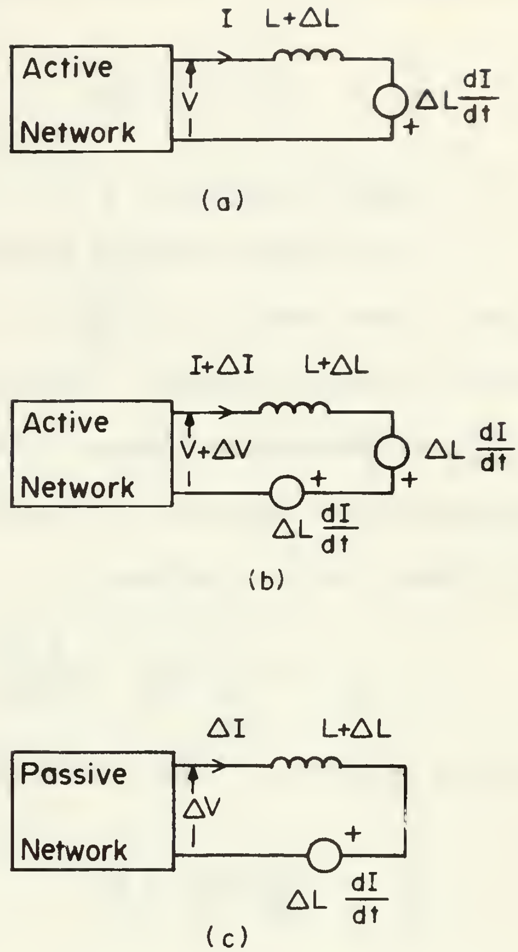


FIG. 2.3 INDUCTIVE CHANGES.

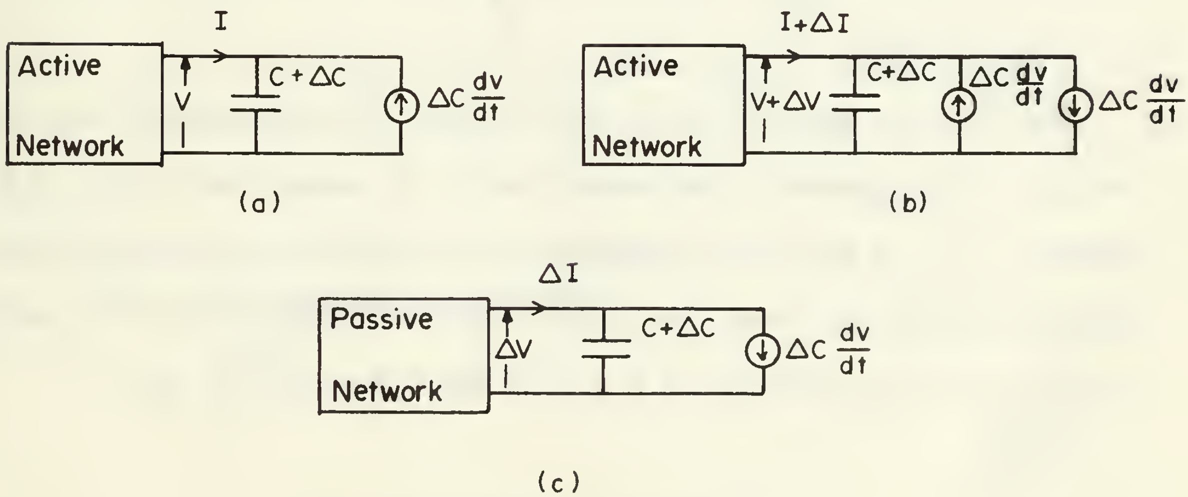


FIG. 2.4 CAPACITIVE CHANGES.

Dividing by ΔL and letting $\Delta L \to 0$ results in (2.12).

$$\frac{\partial V}{\partial L} = L \frac{d}{dt}\left(\frac{\partial I}{\partial L}\right) + \frac{dI}{dt} \tag{2.12}$$

3. Capacitive Change

If the capacitance in a branch is changed by an amount ΔC, and a current source equal to $\Delta C \cdot \left(\frac{dV}{dt}\right)$ is introduced (Fig. 2.4a) the network variables remain unchanged. Introducing another current source (Fig. 2.4b) results in incremental changes in the branch voltage and current. Again by applying the superposition theorem (Fig. 2.4c) the incremental voltage and current can be calculated.

$$\Delta I = (C + \Delta C) \frac{d\Delta V}{dt} + \Delta C \frac{dV}{dt} \tag{2.13}$$

Dividing (2.13) by ΔC and letting $\Delta C \to 0$ results in (2.14).

$$\frac{\partial I}{\partial C} = C \frac{d}{dt}\left(\frac{\partial V}{\partial C}\right) + \frac{dV}{dt} \tag{2.14}$$

Equations (2.10), (2.12), and (2.14) are identical to the auxiliary network equations of Fig. 2.1. This illustrates the direct relationship between the compensation theorem and sensitivity analysis.

D. LINEAR EXAMPLE

Consider the peaking circuit [12] in Fig. 2.5a operating in the linear region. The incremental equivalent circuit is represented in Fig. 2.5b where $R_1 = r_p$ is the plate resistance and C is the coil capacitance plus output capacitance, stray capacitance and wiring capacitance. The equivalent circuit is indicated in Fig. 2.5c where $R = \frac{R_1 R_2}{R_1 + R_2}$ and $v = -\frac{R_1 R_2}{R_1 + R_2} \mu v_i$.

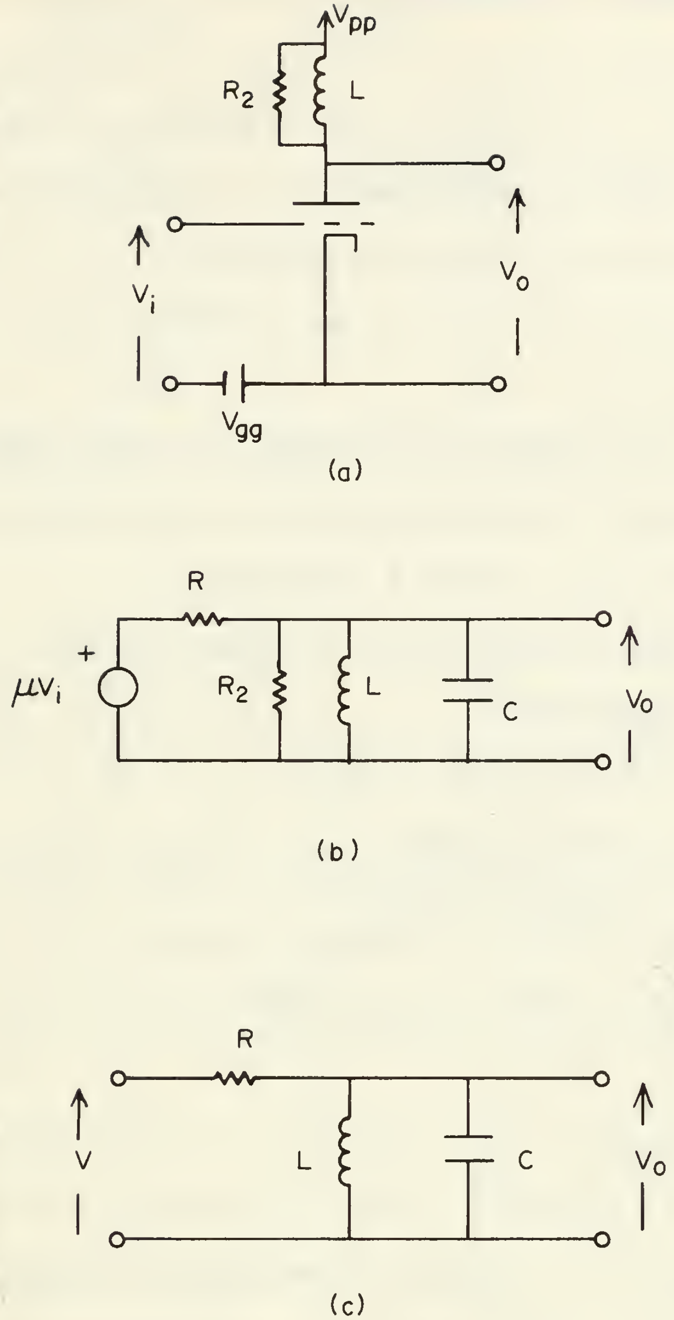


FIG. 2.5 PEAKING CIRCUIT.

It is of interest to determine how sensitive the output is with respect to incremental changes in R, L and C. As representative values let $L = 0.1$ henry, $C = 250\mu$farads, and R be adjustable according to the type of response desired. The response can be determined from the differential equations of the circuit and is underdamped for $K < 1$ ($R = 20$), critically damped for $K = 1$ ($R = 10$), and overdamped for $K > 1$ ($R = 5$); where $K = \frac{1}{2R}\sqrt{L/C}$.

The sensitivity functions may be obtained by constructing auxiliary networks as outlined in the preceding section, or by differentiating (2.1) with respect to ln R, ln L, and ln C respectively.

The state-variable representation is illustrated below. The solution takes the form of (2.8) where

$$\underline{u} = v = \text{input voltage of 70 volts} \tag{2.15a}$$

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \text{output voltage} \\ \text{inductor current} \end{bmatrix} \tag{2.15b}$$

$$A = \begin{bmatrix} -1/RC & -1/C \\ 1/L & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1/RC \\ 0 \end{bmatrix} \tag{2.15c}$$

$$\underline{u}_{sj} = \underline{0} \quad \text{for } j = 1,2,3 \tag{2.15d}$$

$$\underline{x}_{s1} = \begin{bmatrix} \frac{\partial x_1}{\partial \ln R} \\ \frac{\partial x_2}{\partial \ln R} \end{bmatrix} = \begin{bmatrix} \text{sensitivity of output with respect to R} \\ \text{sensitivity of inductor current with respect to R} \end{bmatrix} \tag{2.15e}$$

$$\underline{x}_{s2} = \begin{bmatrix} \frac{\partial x_1}{\partial \ln L} \\ \frac{\partial x_2}{\partial \ln L} \end{bmatrix} = \begin{bmatrix} \text{sensitivity of output with respect to L} \\ \text{sensitivity of inductor current with respect to L} \end{bmatrix} \tag{2.15f}$$

$$\underline{x}_{s3} = \begin{bmatrix} \dfrac{\partial x_1}{\partial \ln C} \\ \\ \dfrac{\partial x_2}{\partial \ln C} \end{bmatrix} = \begin{bmatrix} \text{sensitivity of output with respect to } C \\ \\ \text{sensitivity of inductor current with respect to } C \end{bmatrix} \quad (2.15g)$$

$$A_{s1} = \begin{bmatrix} 1/RC & 0 \\ 0 & 0 \end{bmatrix}, \quad A_{s2} = \begin{bmatrix} 0 & 0 \\ -1/L & 0 \end{bmatrix}, \quad A_{s3} = \begin{bmatrix} 1/RC & 1/C \\ 0 & 0 \end{bmatrix} \quad (2.15h)$$

$$B_{s1} = \begin{bmatrix} -1/RC \\ 0 \end{bmatrix}, \quad B_{s2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad B_{s3} = \begin{bmatrix} -1/RC \\ 0 \end{bmatrix} \quad (2.15i)$$

The computer program used to implement the solution is included in Appendix B. This program is general in that it is easily modified to handle linear circuits containing more state variables and sensitivity parameters.

The time response of the output of the circuit and the corresponding sensitivities are presented in Figs. 2.6 through 2.11 for underdamped, critically damped and overdamped responses. These results have been obtained using trapezoidal integration as well as a fourth-order Runge-Kutta integration technique.

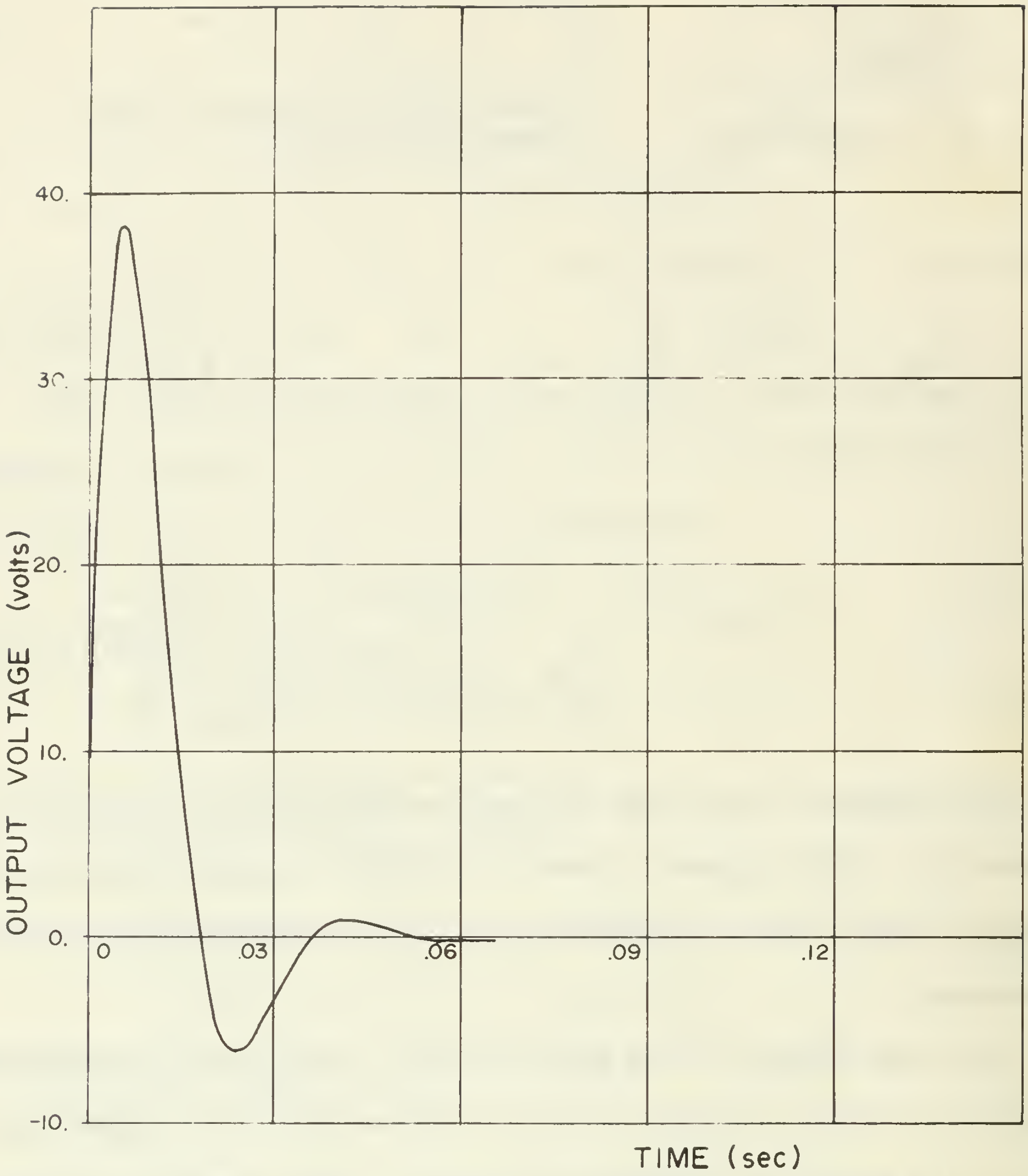


FIG. 2.6. LINEAR EXAMPLE--UNDERDAMPED RESPONSE.

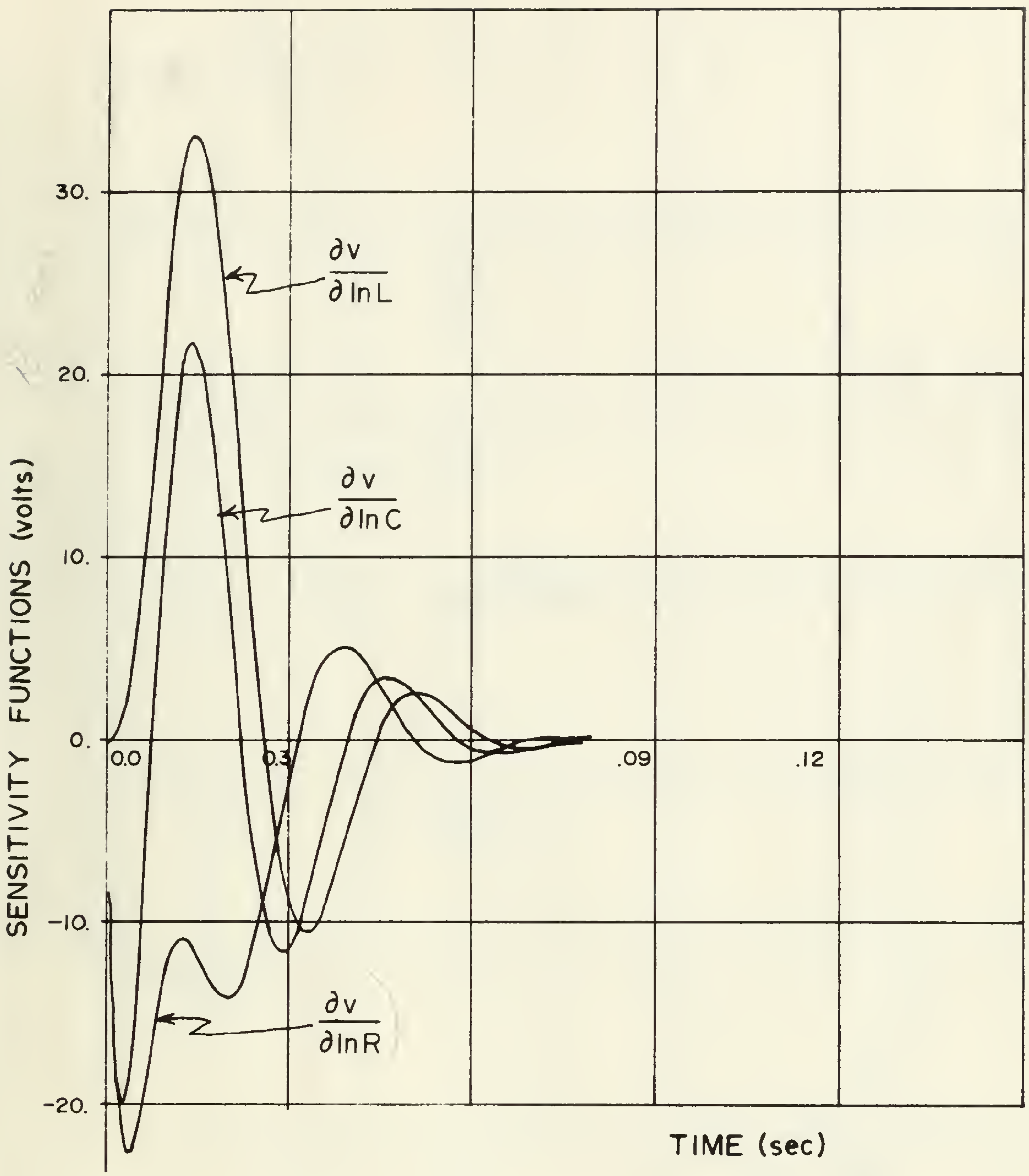


FIG. 2.7. SENSITIVITY FUNCTIONS FOR UNDERDAMPED RESPONSE.

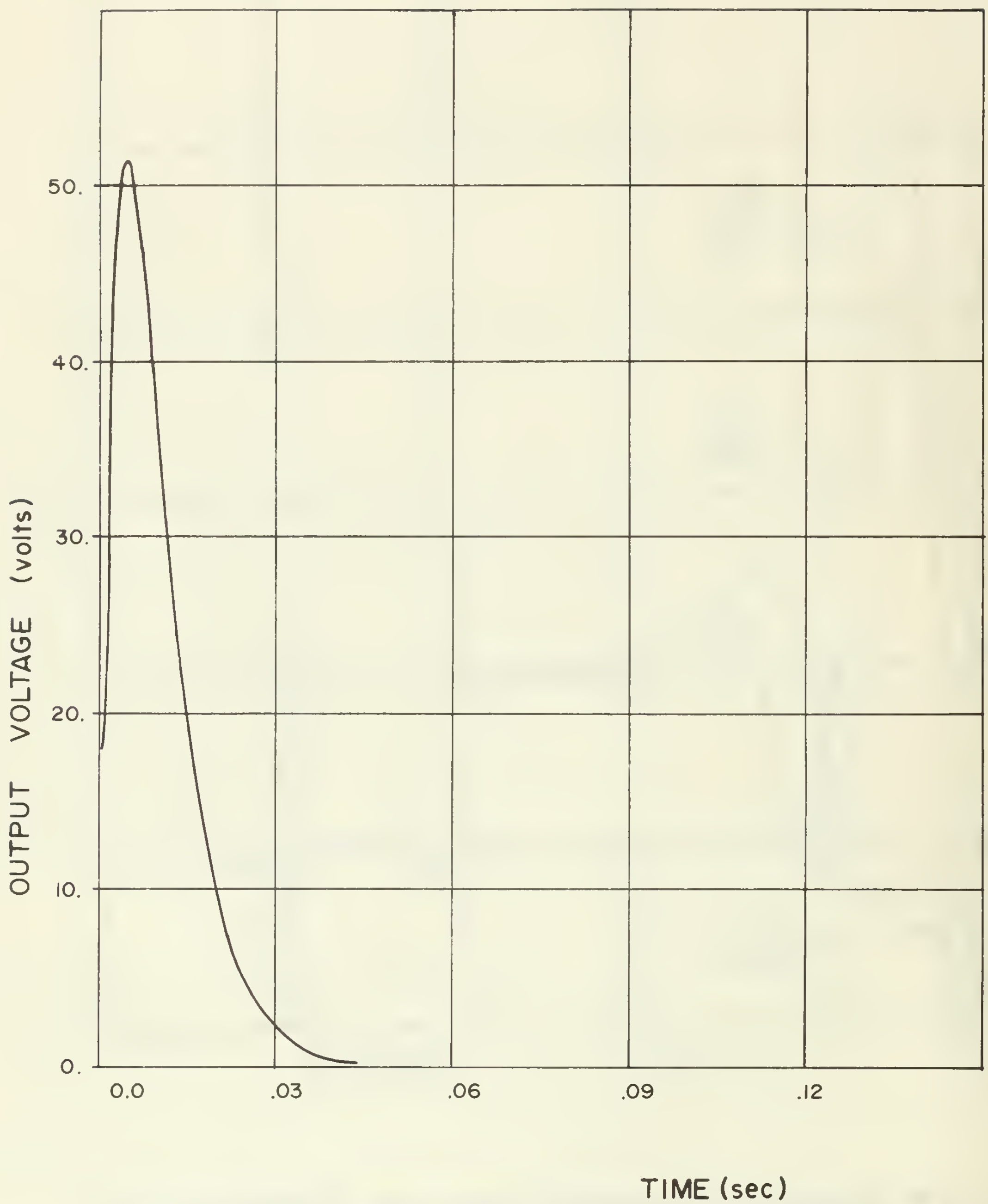


FIG. 2.8. LINEAR EXAMPLE--CRITICALLY DAMPED RESPONSE.

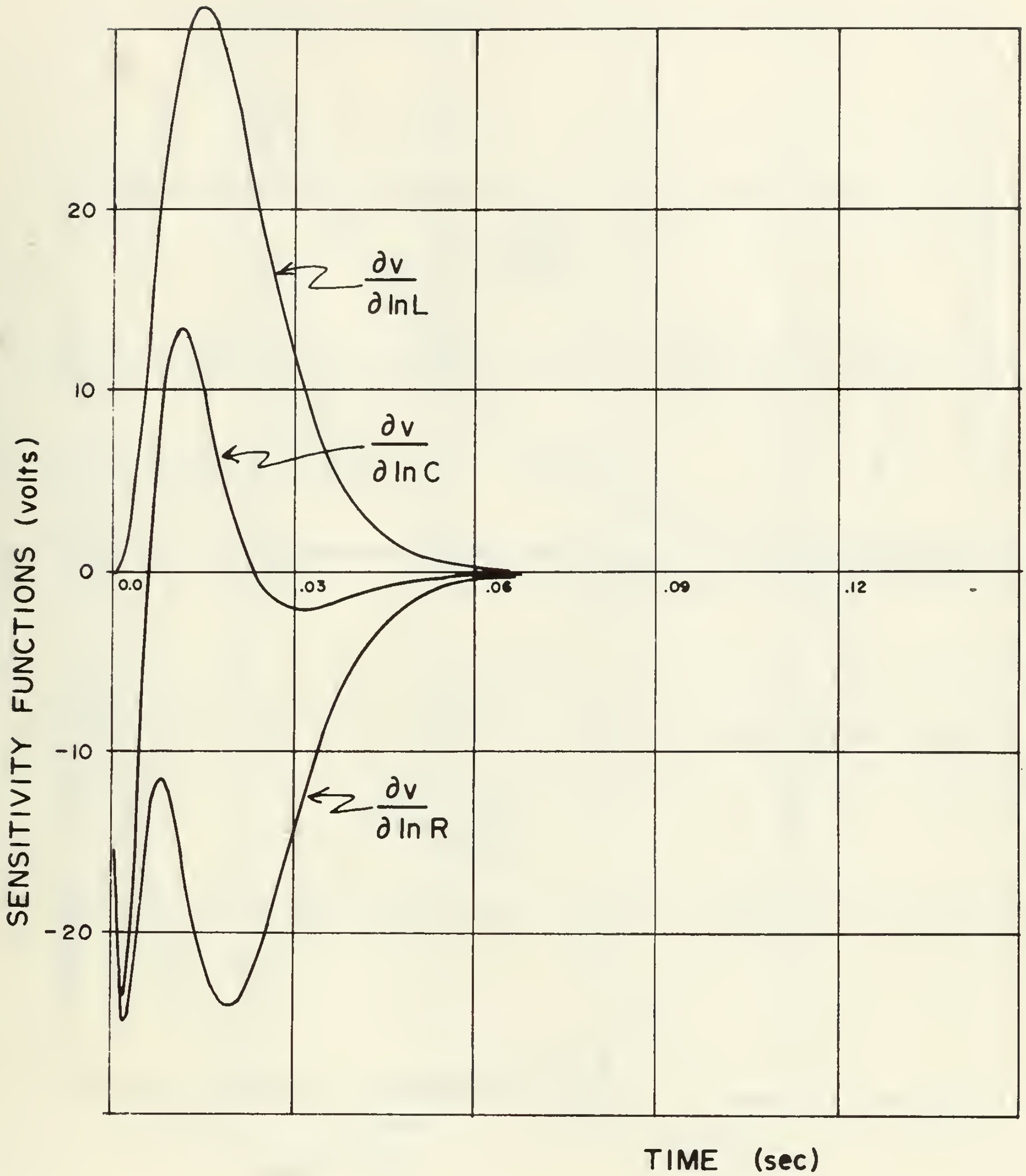


FIG. 2.9. SENSITIVITY FUNCTIONS FOR CRITICALLY DAMPED RESPONSE.

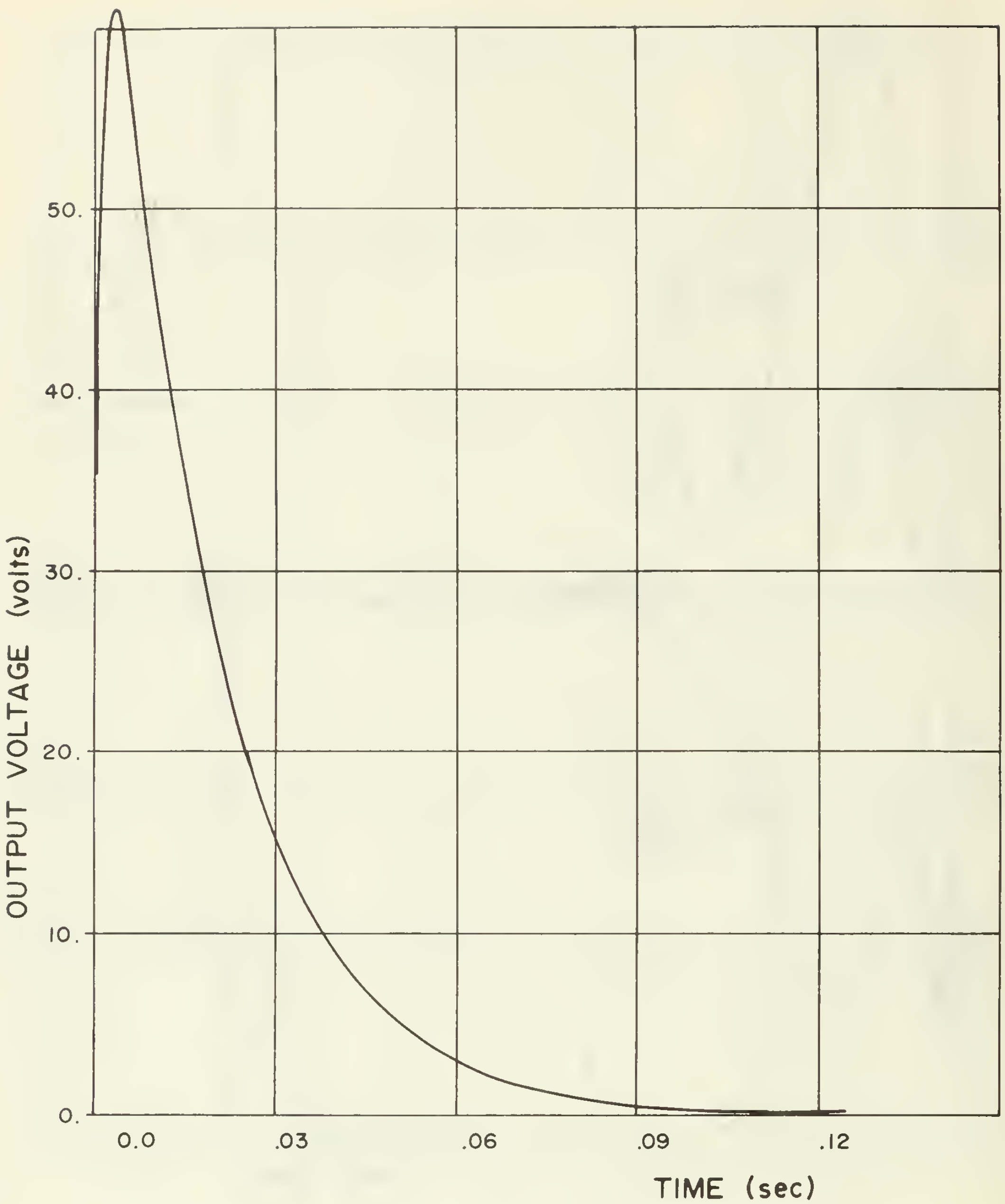


FIG. 2.10. LINEAR EXAMPLE--OVERDAMPED RESPONSE.

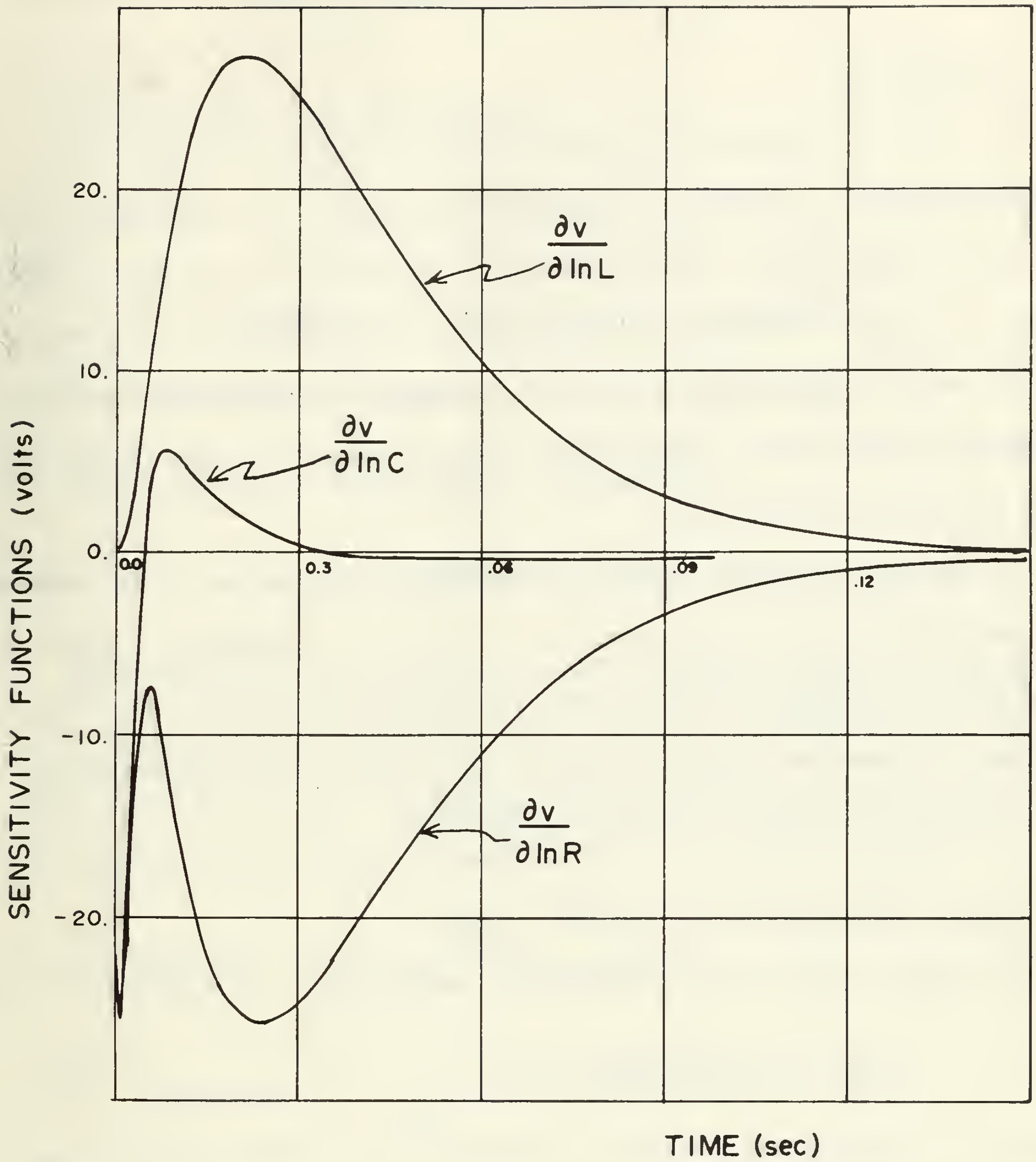


FIG. 2.11. SENSITIVITY FUNCTIONS FOR OVERDAMPED RESPONSE.

III. MODELING OF NONLINEAR CIRCUITS

A. INTRODUCTION

The digital computer has played an increasingly important role in circuit analysis and design in the past ten years. This has been partly due to the emergence of sophisticated circuit-analysis and design programs [13]. In order for these programs to be useful, acceptable nonlinear models which relate model parameters to physical processes are required. These models must lend themselves to numerical analysis and provide acceptable quantitative accuracy.

In the next section, a general mathematical format which is convenient for sensitivity analysis of nonlinear circuits is developed. In the succeeding section modeling of diodes and transistors is outlined. This is followed by examples which consider calculation of the response of a p-n junction to a pulse of radiation current.

B. GENERAL NONLINEAR STATE-VARIABLE FORMAT

The state equation for a nonlinear time-invariant system is of the form

$$\dot{\underline{x}}(t) = \underline{a}(\underline{x}(t),\underline{u}(t)) \tag{3.1}$$

where $\underline{a}$ is a vector of nonlinear functions of the state variables and forcing functions.

As is shown in the section which follows, (3.1) takes the following general form for circuits containing diodes and transistors (represented by the Ebers-Moll [8,14] equations). This representation includes nonlinear resistances and nonlinear capacitances.

$$\dot{\underline{x}}(t) = A(\underline{x}(t))\cdot\underline{x}(t) + \underline{c}(x(t)) + B(\underline{x}(t))\cdot\underline{u}(t) \tag{3.2}$$

where $A(\underline{x}(t))$ is a nxn matrix and $B(\underline{x}(t))$ is an nxm matrix, both with elements which may be nonlinear functions of the states, and $\underline{c}(\underline{x}(t))$ is a general nonlinear vector.

Equation (3.2) may be written as

$$\dot{\underline{x}}(t) = \underline{f}(\underline{x}(t)) + B(\underline{x}(t))\cdot\underline{u}(t) \tag{3.3}$$

where

$$\underline{f}(\underline{x}(t)) = A(\underline{x}(t))\cdot\underline{x}(t) + \underline{c}(\underline{x}(t))$$

Using trapezoidal integration for (3.3) results in

$$\begin{aligned}\underline{x}(n) - \underline{x}(n-1) &= \frac{\Delta T}{2}\,[\underline{f}(\underline{x}(n)) + \underline{f}(\underline{x}(n-1))] \\ &\quad + \frac{\Delta T}{2}\,[B(\underline{x}(n))\cdot\underline{u}(n) + B(\underline{x}(n-1))\cdot\underline{u}(n-1)]\end{aligned} \tag{3.4}$$

Equation (3.4) is a nonlinear matrix equation implicit in $\underline{x}(n)$ which, as a result, is generally difficult to solve. Simplification results if $\underline{f}(\underline{x}(n))$ and $B(\underline{x}(n))\cdot\underline{u}(n)$, where $\underline{u}(n)$ is considered constant, are expanded in a Taylor series about the point $\underline{x}(n-1)$. In matrix notation

$$\underline{f}(\underline{x}(n)) = \sum_{k=0}^{\infty} (\underline{\Delta x}^T\nabla)^k\ \underline{f}(\underline{x}(n-1))/k! \tag{3.5a}$$

$$B(\underline{x}(n))\cdot\underline{u}(n) = \sum_{k=0}^{\infty} (\underline{\Delta x}^T\nabla)^k\ [B(\underline{x}(n-1))\cdot\underline{u}(n)]/k! \tag{3.5b}$$

where

$$\nabla = \left[\frac{\partial}{\partial x_1}\ \frac{\partial}{\partial x_2}\ \cdots\ \frac{\partial}{\partial x_n}\right]^T$$

$\underline{f}$, $B\underline{u}$, $\underline{x}$, $\underline{\Delta x}$ are n vectors

and

$\underline{\Delta x} = \underline{x}(n) - \underline{x}(n-1)$

A first-order approximation for (3.5) results in (3.6).

$$\underline{f}(\underline{x}(n)) = \underline{f}(\underline{x}(n-1)) + (\underline{\Delta x}^T\nabla)\ \underline{f}(\underline{x}(n-1)) \tag{3.6a}$$

$$B(\underline{x}(n))\cdot\underline{u}(n) = B(\underline{x}(n-1))\cdot\underline{u}(n) + (\underline{\Delta x}^T\nabla)\ [B(\underline{x}(n-1))\cdot\underline{u}(n)] \tag{3.6b}$$

Substituting (3.6) into (3.4) yields

$$\underline{x}(n) - \underline{x}(n-1) = \frac{\Delta T}{2}\left[\ 2\underline{f}(\underline{x}(n-1)) + (\underline{\Delta x}^T\nabla)\ \underline{f}(\underline{x}(n-1))\ \right]$$

$$+ \frac{\Delta T}{2}\left[B(\underline{x}(n-1)) \cdot (\underline{u}(n) + u(n-1)) + (\underline{\Delta x}^T\nabla)\ \{B(\underline{x}(n-1))\cdot\underline{u}(n)\}\right] \tag{3.7}$$

Equation (3.7) may be simplified by using the matrix identity

$$(\underline{\Delta x}^T\nabla)\ \underline{f} = (\nabla\underline{f}^T)^T\ \underline{\Delta x} \tag{3.8}$$

which is proven in Appendix A.

Substituting (3.8) into (3.7) and solving for $\underline{x}(n)$ results in

$$\underline{x}(n) = \underline{x}(n-1) + \Delta T\cdot\left[I - \frac{\Delta T}{2}[(\nabla\underline{f}^T)^T + \{\nabla[B\cdot\underline{u}(n)]^T\}^T]\right]^{-1}$$

$$\left[\underline{f} + \frac{1}{2} B \cdot (\underline{u}(n) + \underline{u}(n-1))\right] \tag{3.9}$$

where $\underline{f}$, B, $\nabla\underline{f}^T$ and $(B\cdot\underline{u}(n))^T$ are evaluated at the point $\underline{x}(n-1)$. Equation (3.9) can be solved directly for $\underline{x}(n)$ from the known values of $\underline{x}(n-1)$, $\underline{u}(n)$, and $\underline{u}(n-1)$.

C. DIODE AND TRANSISTOR MODELING

1. Modeling for a p-n Junction Diode

Diode modeling has received much attention during the past twenty years [15,16]. Recently Steele [17] prepared a report concerning semiconductor modeling specifically for computer analysis. The applicable results are summarized in this section.

a. Current-Voltage Relationship

The current-voltage relationship for an ideal diode is

$$i_d = I_S (e^{\alpha V} - 1) \tag{3.10}$$

where I_S = reverse saturation current

α = a parameter dependent upon the temperature and the type of semiconductor material.

Generally the reverse saturation current is not entirely independent of voltage. This is due to a leakage current and can be accounted for by the addition of a resistance, R_{SH}, shunting the diode. Also, at large reverse voltages an effect known as Zener breakdown occurs. This effect is omitted in the following analysis.

b. Diode Resistance

The static resistance $R_d(v)$ of a diode is defined as the ratio v/i_d. Since $R_d(v)$ varies widely with v and i_d, it is generally not a useful parameter. An incremental, or dynamic, resistance $r_d(v)$ is defined as $r_d(v) = \frac{dv}{di_d} = \frac{1}{I_S \alpha e^{\alpha V}}$. Although the dynamic resistance varies widely with v it is a useful parameter for small-signal operation.

c. Capacitance Effect

Diode capacitance is due to two separate effects; space charge and charge storage.

(1) Space Charge. A diffusion potential, V_D, is established in the junction region when p and n materials are placed in contact. This is caused by an increase in donor atoms on the n side and acceptor atoms on the p side of the junction, resulting in a capacitive effect. As a reverse voltage is applied more donor and acceptor atoms are uncovered causing the junction width to expand, and hence the capacitance to decrease. This voltage-dependent capacitance has the following form when the diode

is reverse biased. For radiation analysis, acceptable results are obtained when C_D is assumed to be constant.

$$C_D = \frac{C_0}{(1 - v/v_D)^n} \tag{3.11}$$

where $n = 1/2$ for abrupt junctions

$n = 1/3$ for linear-graded junctions.

(2) Charge Storage. When the diode junction is forward biased the potential barrier is lowered. This results in a high density of minority carriers injected into the p and n regions, which can be viewed as stored carriers. A change in voltage results in a change in density (number of stored carriers) and hence a capacitive effect.

If the voltage on the junction is abruptly reversed the stored carriers reach equilibrium after some storage time T_D. The associated current may be defined as

$$i = T_D \frac{di_d}{dt} \tag{3.12}$$

Substituting (3.10) into (3.12) yields

$$i = T_D I_S \alpha e^{\alpha v} = C_S(v) \frac{dv}{dt} \tag{3.13}$$

where $C_S(v) = I_S \alpha e^{\alpha v}$ is the charge-storage capacitance. The charge-storage capacitance may be alternately written in terms of the dynamic resistance as $C_S(v) = T_D/r_d(v)$.

d. Diode Model

Combining the effects outlined above results in the diode model in Fig. 3.1. The circuit equation becomes

$$C_d(v) \frac{dv}{dt} + i_d + \frac{v}{R_{SH}} = 0 \tag{3.14}$$

where $C_d(v) = C_D + C_S(v)$.

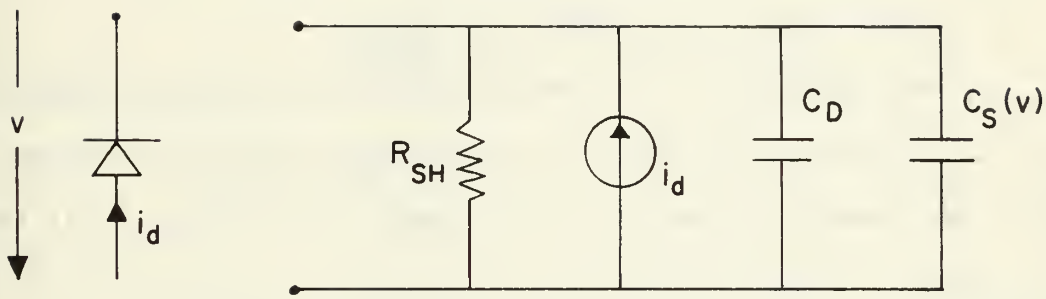


Fig. 3.1. Diode model.

Equation (3.14) is in the form of (3.3) where

$$\underline{f}(\underline{x}(t)) = -\frac{i_d}{C_d(v)} - \frac{v}{C_d(v)\cdot R_{SH}} \tag{3.15a}$$

$$B(\underline{x}(t)) = 0 \tag{3.15b}$$

The diode example (III. D.) does not include parameter temperature dependence. The range of values studied includes a spread in parameter values as may be due to manufacturing tolerances, environmental conditions, and aging. The Zener breakdown effect and voltage dependence of C_D are neglected. Typical parameter values are used.

2. Modeling for a p-n Junction Transistor

Many models for a p-n junction transistor have been proposed in the last fifteen years, the most common being the Ebers-Moll model [8,14], the lumped model [16], and the charge-control model [18,19]. It has been shown that each model yields the same result in the solution of a large-signal transient problem [20]. The Ebers-Moll model is used in this study and is summarized in this section.

a. Current-Voltage Relationship

The current-voltage equations for the Ebers-Moll model are given in (3.16) and (3.17).

$$I_E = \frac{\alpha_I I_{CO}}{1 - \alpha_I \alpha_N}\left(e^{\alpha V_C} - 1\right) - \frac{I_{EO}}{1 - \alpha_I \alpha_N}\left(e^{\alpha V_E} - 1\right) \tag{3.16}$$

$$I_C = \frac{\alpha_N I_{EO}}{1 - \alpha_I \alpha_N}\left(e^{\alpha V_E} - 1\right) - \frac{I_{CO}}{1 - \alpha_I \alpha_N}\left(e^{\alpha V_C} - 1\right) \tag{3.17}$$

where

α = constant having a value between 1 and 2

α_N = common-base current in the normal mode

α_I = common-base current gain in the inverse mode

I_{CO} = saturation current of collector junction with zero emitter current

I_{EO} = saturation current of emitter junction with zero collector current

b. Transistor Model

Figs. 3.2a and 3.3a are representations of (3.16) and (3.17) for a n-p-n and p-n-p transistor respectively. Figs. 3.2b and 3.3b represent the corresponding equivalent circuits if each diode is modeled as outlined in the preceding section.

The reverse saturation currents are as follows:

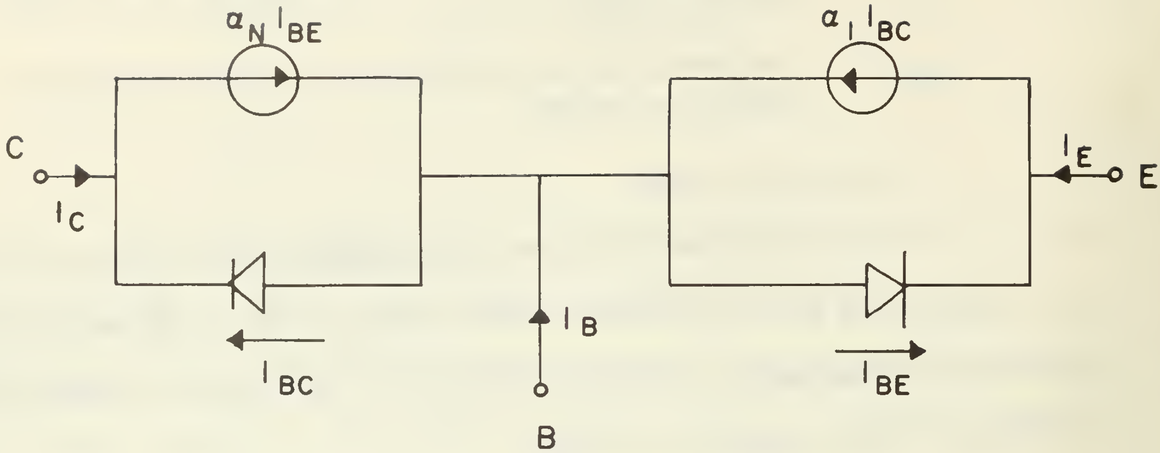
$$I_{CS} = \frac{\alpha_I I_{CO}}{1 - \alpha_I \alpha_N} \tag{3.18a}$$

$$I_{ES} = \frac{\alpha_N I_{EO}}{1 - \alpha_I \alpha_N} \tag{3.18b}$$

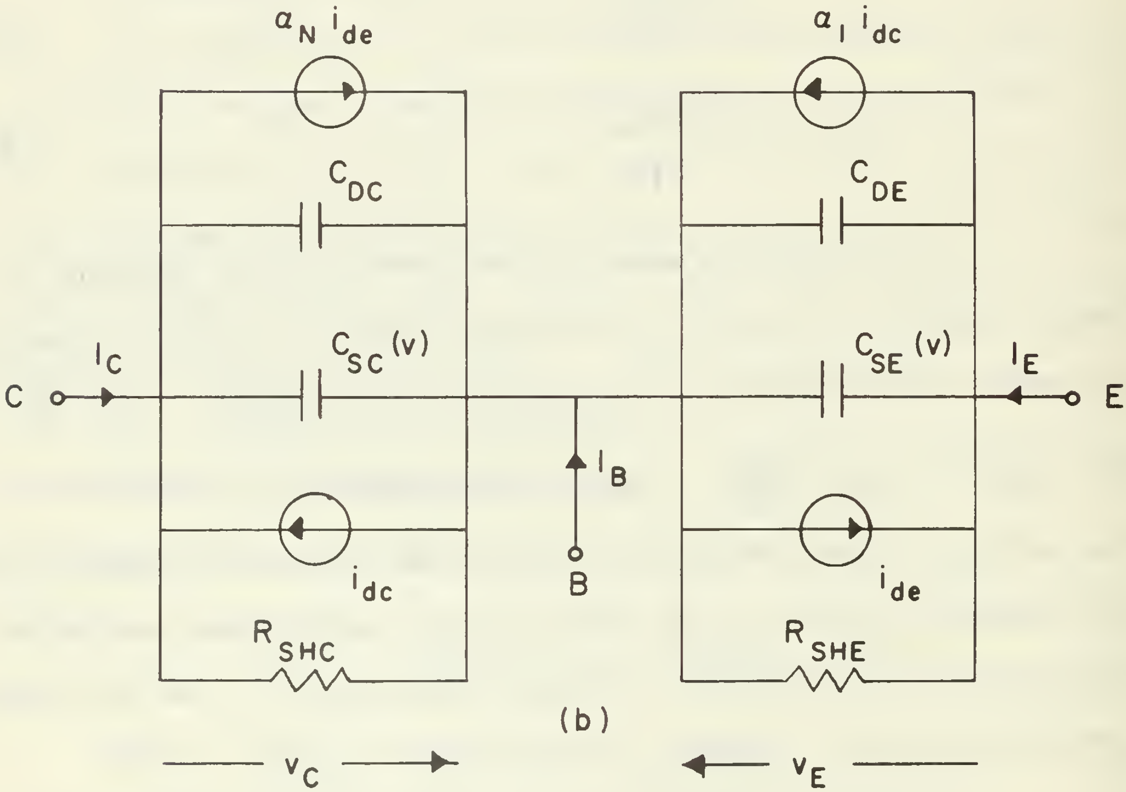
If the positive senses of the currents are as indicated in Figs. 3.2 and 3.3, I_{EO} and I_{CO} are positive quantities for n-p-n transistors and negative quantities for p-n-p transistors and $C_{SC}(v) = T_{DC} \frac{di_{dc}}{dt}$ and $C_{SE}(v) = T_{DE} \frac{di_{de}}{dt}$. T_{DE} is approximated by the transistor rise time and T_{DC} is approximated by the sum of the transistor storage time and the transistor fall time. The rise, fall and storage times are complex functions of the base current, collector current, α_N, α_I, and the transistor current-gain cut-off frequency, and are defined in Fig. 3.4 [17].

If $C_{dc}(v) = C_{DC} + C_{SC}(v)$ and $C_{de}(v) = C_{DE} + C_{SE}(v)$ the equations for Fig. 3.2b become

$$C_{dc}(v) \frac{dv_C}{dt} + i_{dc} + \frac{v_C}{R_{SHC}} - \alpha_N i_{de} = 0 \tag{3.19a}$$



(a)



(b)

Fig. 3.2. Model and equivalent circuit for the n-p-n transistor.

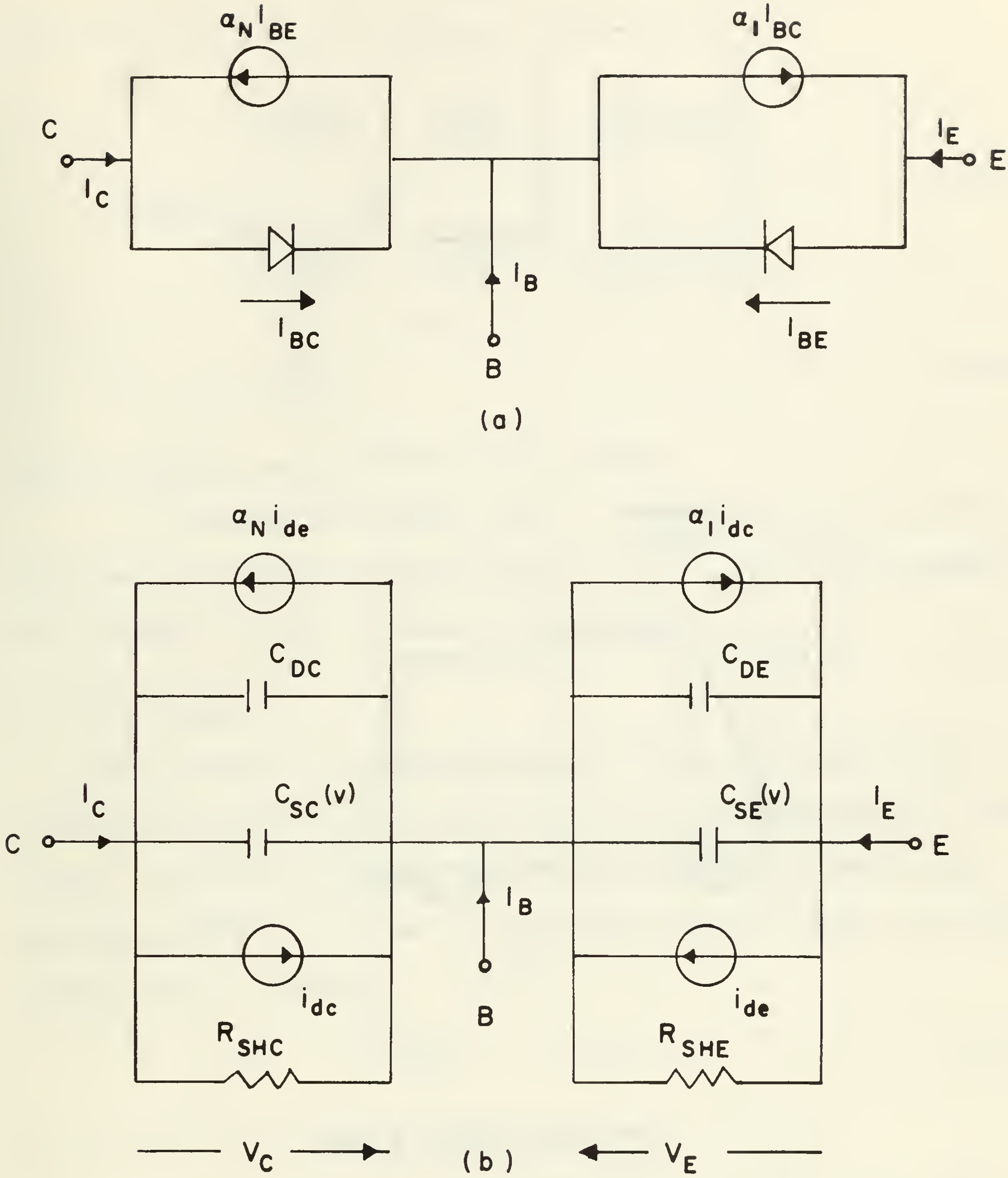


Fig. 3.3. Model and equivalent circuit for the p-n-p transistor.

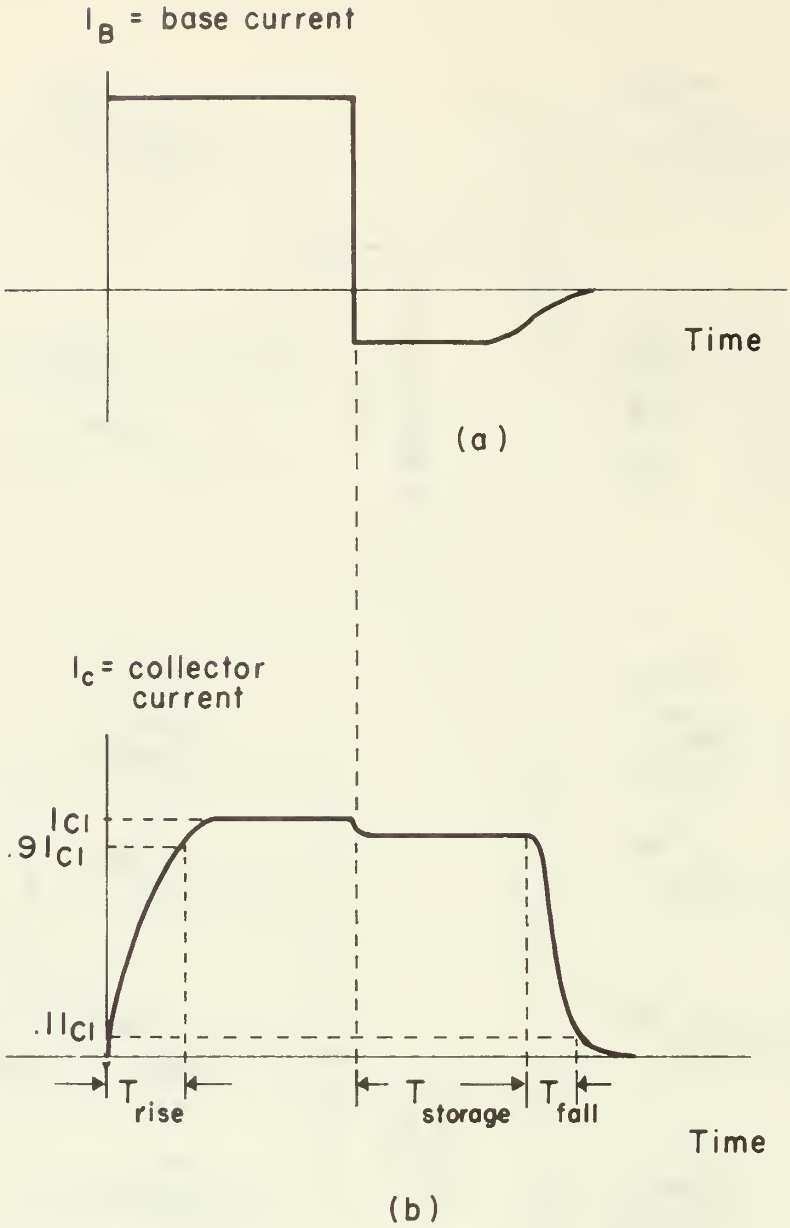


Fig. 3.4. Transistor switching times.

$$C_{de}(v)\,\frac{dv_E}{dt} + i_{de} + \frac{v_E}{R_{SHE}} - \alpha_I i_{dc} = 0 \qquad (3.19b)$$

which are in the form of (3.3) where

$$\underline{f}(\underline{x}(t)) = \begin{bmatrix} \left(\dfrac{\alpha_N i_{de}}{C_{dc}(v)} - \dfrac{i_{dc}}{C_{dc}(v)} - \dfrac{v_C}{R_{SHC}C_{dc}(v)}\right) \\ \\ \left(\dfrac{\alpha_I i_{dc}}{C_{de}(v)} - \dfrac{i_{de}}{C_{de}(v)} - \dfrac{v_E}{R_{SHE}C_{de}(v)}\right) \end{bmatrix} \qquad (3.20a)$$

$$B = [0] \qquad (3.20b)$$

Similar equations can be written for Fig. (3.3b)

The transistor example (III. E.) does not include the effects of parameter temperature dependence and C_{DE}, C_{DC}, T_{DE} and T_{DC} are assumed to be constant. Typical parameter values are used.

D. DIODE EXAMPLE

A diode problem of considerable interest is now solved in order to illustrate the modeling and the mathematical formulation discussed in this chapter. The problem considers the effect of a radiation current pulse injected across a p-n junction as indicated in Fig. 3.5, where the radiation current pulse is defined by

$$i_{pp}(t) + T_R\,\frac{di_{pp}(t)}{dt} = A\,\gamma(t) \qquad (3.21)$$

where $\gamma(t)$ is the unit pulse given in Fig. 3.6.

Replacing the diode in Fig. 3.5 by the model described above yields Fig. 3.7. Typical values are

$I_S = 1.0$ pA	$C_D = 1.0$ pF	$T_D = 1.0\mu s$
$\alpha = 1/.026\ V^{-1}$	$R_{SH} = 100.0\ M\Omega$	$T_R = 0.2\mu s$
$A = 10.0$ mA		

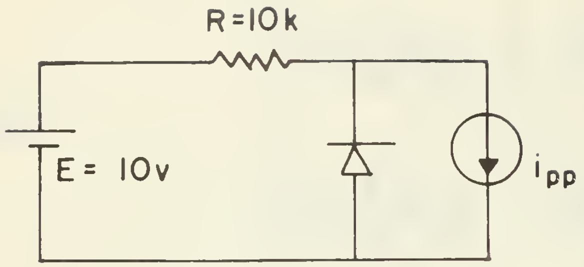


Fig. 3.5. Diode circuit.

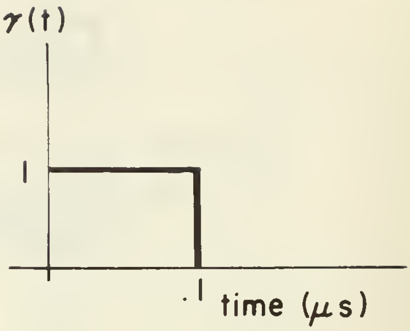


Fig. 3.6. Radiation current pulse.

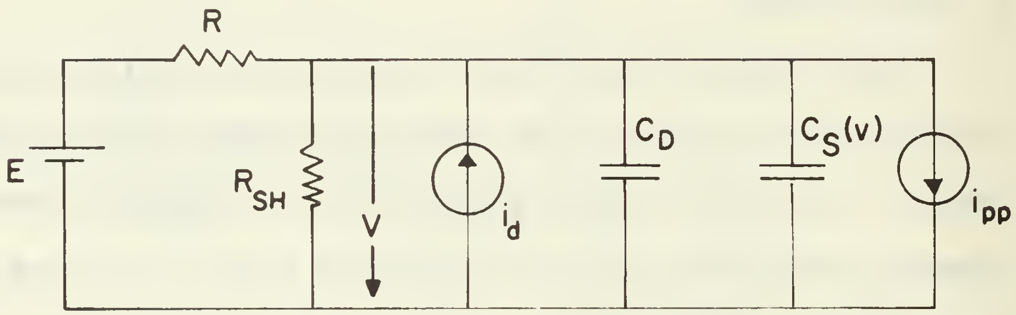


Fig. 3.7. Equivalent circuit.

1. Analytical Solution for Radiation Current

Equation (3.21) can be solved using Laplace transforms.

$$I_{pp}(s) = \begin{cases} A\cdot\left(\dfrac{1}{s} - \dfrac{1}{s + 1/T_R}\right) & 0 < t < 10^{-7} \\ .394\ A\cdot\left(\dfrac{1}{s + 1/T_R}\right) & t \geq 10^{-7} \end{cases} \tag{3.22}$$

The resulting time response is given in (3.23).

$$i_{pp}(t) = \begin{cases} A\cdot(1 - e^{-t/T_R}) & 0 < t < 10^{-7} \\ .394\ A\ e^{-(t-10^{-7})/T_R} & t \geq 10^{-7} \end{cases} \tag{3.23}$$

2. Piecewise-Linear Diode Model

If the diode current-voltage relationship is assumed to be piecewise linear (Fig. 3.8) $i_d = 0$ for $v < 0$ and $v = i_d R_d$ for $v > 0$, where $R_d = R_d(v) = r_d(v)$. The storage capacitance becomes a constant, $C_S = T_D/R_d$, for $v > 0$ and an open circuit for $v < 0$. Fig. 3.7 can be redrawn as in Fig. 3.9 where R_T is the parallel combination of R, R_{SH} and R_d and $C = C_S$ for $v > 0$, and R_T is the parallel combination of R and R_{SH} and $C = C_D$ for $v < 0$.

The rise and fall time can be determined using Laplace transforms.

For $0 < t < 10^{-7}$

$$Z(s) = \frac{1/C}{s + 1/RC} \tag{3.24a}$$

$$\begin{aligned} I(s) &= I_{pp}(s) - \frac{E/R}{s} \\ &= A\cdot\left(\frac{1}{s} - \frac{1}{s + 1/T_R}\right) - \frac{E/R}{s} \end{aligned} \tag{3.24b}$$

The voltage takes the form

$$v(t) = C_1 + C_2 e^{-t/T_R} + C_3 e^{-t/RC} \qquad 0 < t < 10^{-7} \tag{3.25}$$

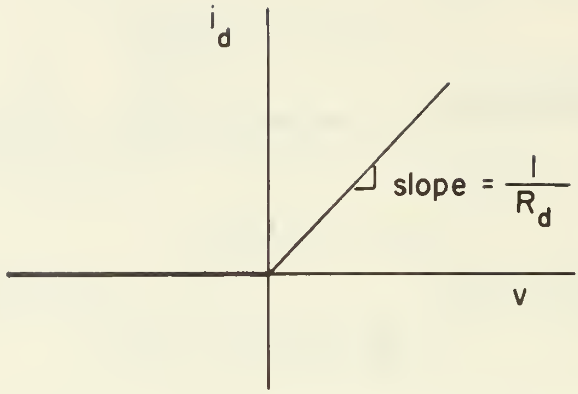


Fig. 3.8. Piecewise linear model.

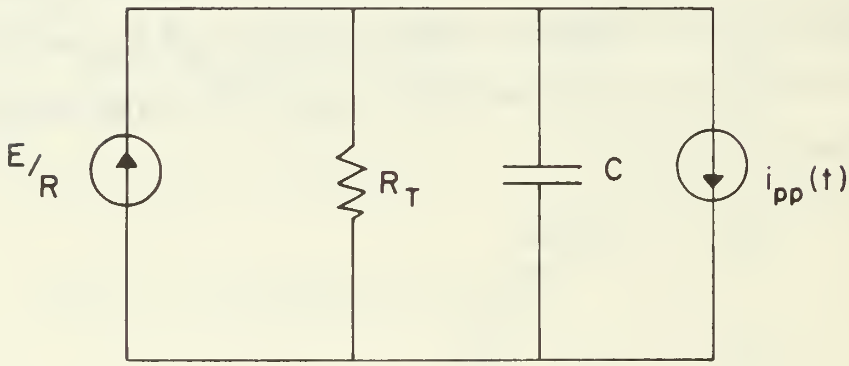


Fig. 3.9. Diode circuit representation.

For $t \geq 10^{-7}$

$$I_S = .394\ A \cdot \left(\frac{1}{s - 1/T_R}\right) - \frac{E/R}{s} \tag{3.26}$$

The resulting voltage response is given by

$$v(t) = C_4 + C_5 e^{-t/T_R} + C_6 e^{-t/RC} \qquad t \geq 10^{-7} \tag{3.27}$$

The differential equation becomes

$$C \frac{dv}{dt} = i_{pp} - Gv - E/R \tag{3.28}$$

where $G = 1/R_T$.

Equation (3.28) is solved using trapezoidal integration. Several values of R_d are used and the output voltage, $v_o = -v$, is indicated in Fig. 3.10. The computer program is presented in Appendix B. The solution in Fig. 3.10 is of little practical value because of the wide variation of the voltage response with assumed values of R_d.

3. Taylor Series Approximation

Equation (3.28) may be rewritten as

$$C(v)\ dv = i_{pp}\ dt - G(v) \cdot v\ dt - E/R\ dt \tag{3.29}$$

where $C(v) = C_D + C_S(v)$

$$G(v) = \frac{1}{R} + \frac{1}{R_{SH}} + \frac{1}{R_d(v)}$$

Using trapezoidal integration for (3.29) yields

$$\begin{aligned}[C(v(n)) + C(v(n-1))] \frac{\Delta v}{2} &= \frac{\Delta T}{2} [i_{pp}(n) + i_{pp}(n-1) - \frac{2E}{R}] \\ &- \frac{\Delta T}{2} [G(v(n)) \cdot v(n) + G(v(n-1)) \cdot v(n-1)]\end{aligned} \tag{3.30}$$

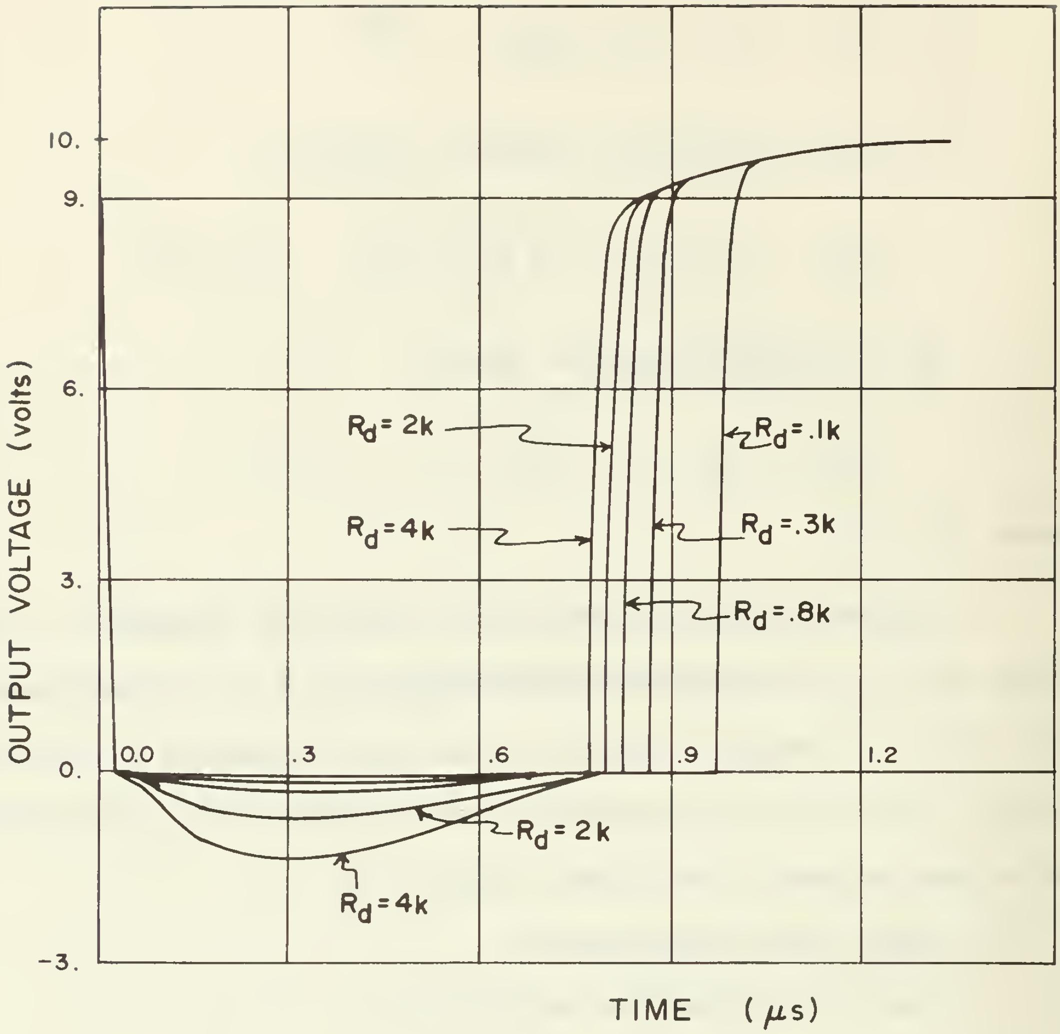


FIG. 3.10. NONLINEAR EXAMPLE--PIECEWISE LINEAR MODEL WITH R_d ADJUSTED FROM 0.1 kΩ TO 4.0 kΩ.

Expanding C(v(n)) and G(v(n))·v(n) in a Taylor series and including only the first-order terms yields

$$C(v(n)) = C(v(n-1)) + \frac{\partial C}{\partial v}\bigg|_{v(n-1)} \cdot \Delta v = C(v(n-1)) \tag{3.31a}$$

$$G(v(n))\cdot v(n) = G(v(n-1))\cdot v(n-1) + G(v(n-1))\cdot\Delta v + \frac{\partial G}{\partial v}\bigg|_{v(n-1)} \cdot \Delta v\cdot v(n-1)$$

$$= G(v(n-1))\cdot v(n) \tag{3.31b}$$

where it is assumed $\frac{\partial C}{\partial v}\Big|_{v(n-1)} = 0$ and $\frac{\partial G}{\partial v}\Big|_{v(n-1)} = 0$.

Substituting (3.31) into (3.30) results in

$$\begin{aligned} C(v(n-1))\cdot\Delta v &= \frac{\Delta T}{2}\left[i_{pp}(n) + i_{pp}(n-1) - \frac{2E}{R}\right] \\ &- \frac{\Delta T}{2}\left[G(v(n-1))\cdot(v(n) - v(n-1))\right] \end{aligned} \tag{3.32}$$

Solving for v(n) yields

$$\begin{aligned} v(n) &= \frac{2C(v(n-1)) - \Delta T\cdot G(v(n-1))}{2C(v(n-1)) + \Delta T\cdot G(v(n-1))}\, v(n-1) \\ &+ \Delta T \cdot \frac{i_{pp}(n) + i_{pp}(n-1) - \frac{2E}{R}}{2C(v(n-1)) + \Delta T \cdot G(v(n-1))} \end{aligned} \tag{3.33}$$

It is of interest to note that the same result is obtained if (3.28) is integrated over the interval (n-1)T to nT as a linear, constant-coefficient equation with C(v) and G(v) having the values C(v(n-1)) and G(v(n-1)) respectively.

This method is implemented with an adjustable integration step size. The step size is adjusted to insure that C(v) and G(v) do not vary by more than a fixed percentage from (n-1)T to nT. The resulting output voltage, v_o, is indicated in Fig. 3.11, where the maximum that C(v) and

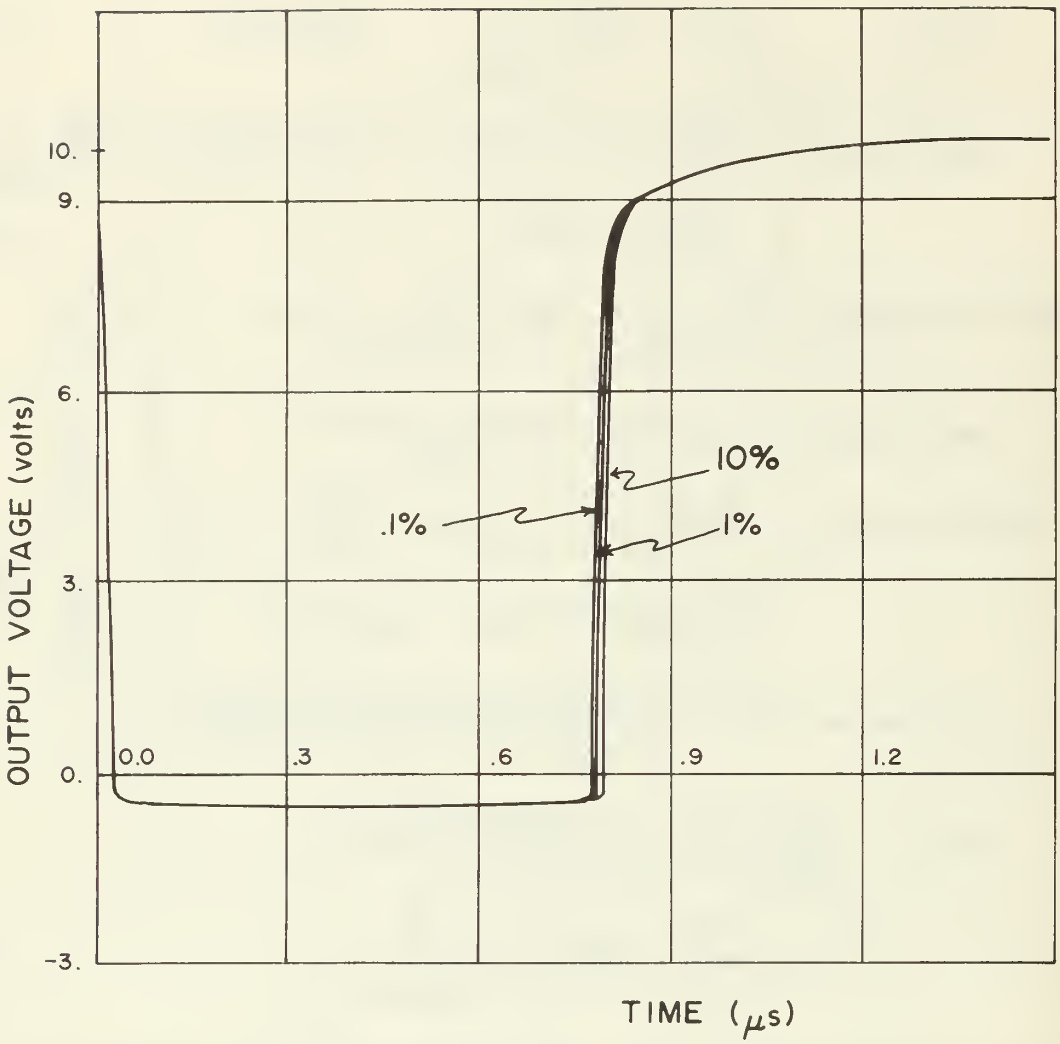


FIG. 3.11. NONLINEAR EXAMPLE -- DIODE RESISTANCE AND CAPACITANCE ARE HELD CONSTANT BETWEEN CALCULATION POINTS. PERCENTAGE VARIATION INDICATES THE MAXIMUM ALLOWED VARIATION IN C(v) AND G(v) BETWEEN CALCULATION POINTS.

G(v) are allowed to vary is 10%, 1% and 0.1%. The computer program is presented in Appendix B. This method becomes more accurate as the percentage variation allowed for C(v) and G(v) is reduced.

4. Diode Solution using General Nonlinear State-Variable Format

The circuit in Fig. 3.7 contains a single state variable, v(t). The state equation for the circuit can be represented in the form of (3.3) where

$$f(v(t)) = \frac{-G \cdot v(t) - I_S\,(e^{\alpha v(t)} - 1)}{C_D + T_D I_S \alpha e^{\alpha v(t)}} \tag{3.34a}$$

$$B(v(t)) = \frac{1}{C_D + T_D I_S \alpha e^{\alpha v(t)}} \tag{3.34b}$$

$$u(t) = i_{pp}(t) - E/R \tag{3.34c}$$

where $G = \frac{1}{R} + \frac{1}{R_{SH}}$.

For the single state-variable problem considered in this example (3.9) reduces to

$$v(u) = v(n-1) + \frac{\Delta T\,[f + \frac{1}{2} B \cdot (u(n) + u(n-1))]}{1 - \frac{\Delta T}{2}\,[\frac{\partial f}{\partial v} + \frac{\partial (B \cdot u(n))}{\partial v}]} \tag{3.35}$$

where the partial derivatives may be obtained from (3.34) and f, B, $\frac{\partial f}{\partial v}$, and $\frac{\partial (B \cdot u(n))}{\partial v}$ are evaluated at v(n-1).

The computer program for this method appears in Appendix B and the results are indicated in Fig. 3.12. The program adjusts ΔT automatically so that Δv never exceeds 0.01 volts. These results have been confirmed by direct integration of the nonlinear state equation using a fourth-order Runge-Kutta integration scheme.

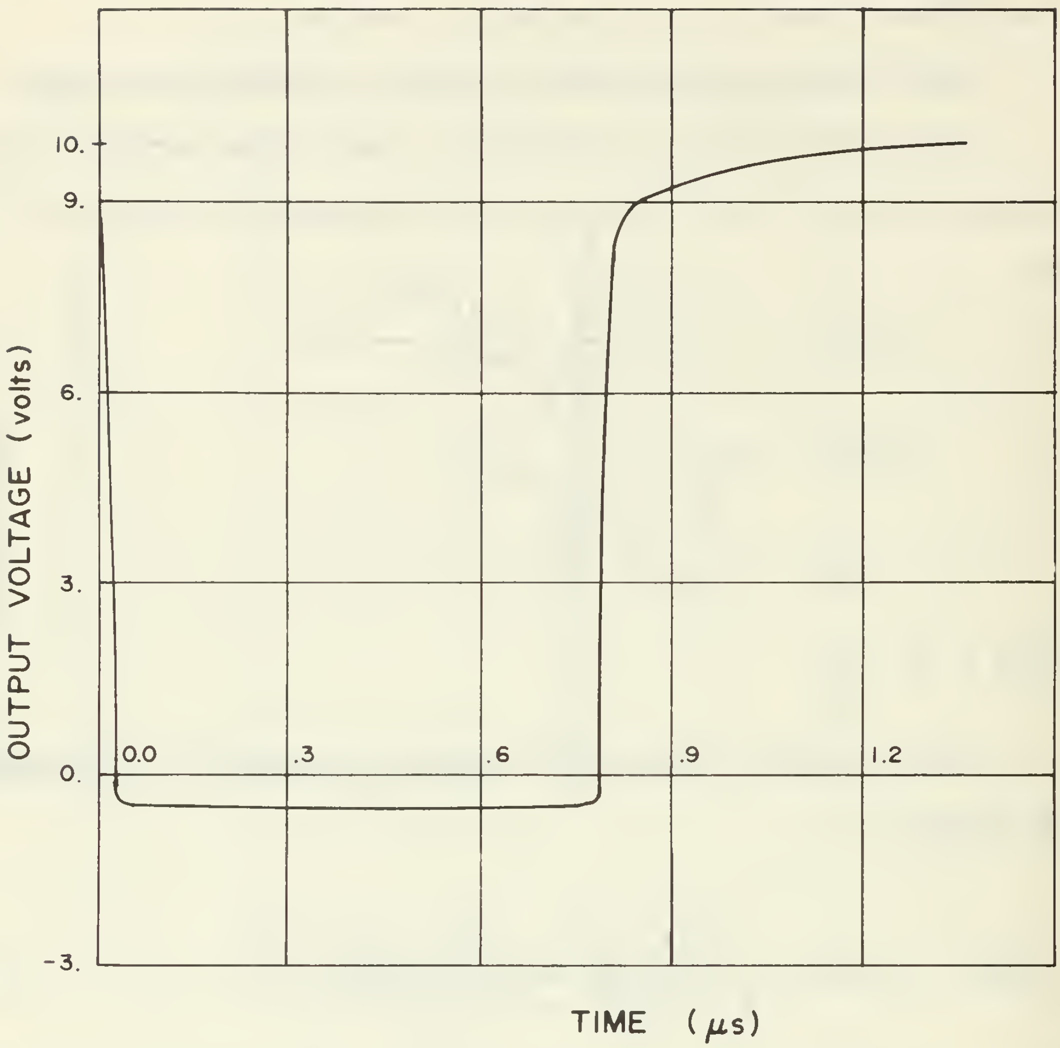


FIG. 3.12. NONLINEAR EXAMPLE -- SOLUTION USING TAYLOR SERIES METHOD.

5. Presentation of Data

The results of several runs are presented in Figs. 3.13 through 3.17. The recovery time, defined as the time for the voltage to recover to 90% of its original value, is plotted as the ordinate. The abscissa represents the parameter value.

In Fig. 3.13 the amplitude of the radiation pulse, A, is varied from 0.265 mA to 20.0 mA. When A $\leq$ 0.265mA the radiation current pulse does not cause the voltage to drop below the 90% value.

In Figs. 3.14 through 3.17 C_D, R, T_R, and T_D are adjusted with A = 5.0 mA and A = 10.0 mA respectively while the other circuit parameters are held constant at their nominal values.

E. TRANSISTOR EXAMPLE

The effect of a radiation current pulse injected across the base-emitter p-n junction of a transistor operating in the active region (Fig. 3.18) is considered in this section. The radiation current pulse is defined by (3.21) where $\gamma(t)$ is the unit pulse given in Fig. 3.19.

1. Transistor Solution using General Nonlinear State-Variable Format

Modeling the transistor in Fig. 3.18 by the equivalent circuit presented in Fig. 3.2 results in Fig. 3.20 where

$$C_{SC}(v) = T_{DC}\, I_{CS}\, \alpha e^{\alpha v_C} \tag{3.36a}$$

$$C_{SE}(v) = T_{DE}\, I_{ES}\, \alpha e^{\alpha v_E} \tag{3.36b}$$

$$i_{de} = I_{ES}\,(e^{\alpha v_E} - 1) \tag{3.36c}$$

$$i_{dc} = I_{CS}\,(e^{\alpha v_C} - 1) \tag{3.36d}$$

Typical parameter values used are

$\alpha_N = 1.0$ $\qquad$ $I_{SC} = 0.01\text{nA}$ $\qquad$ $T_{DE} = 1.0\text{nsec}$

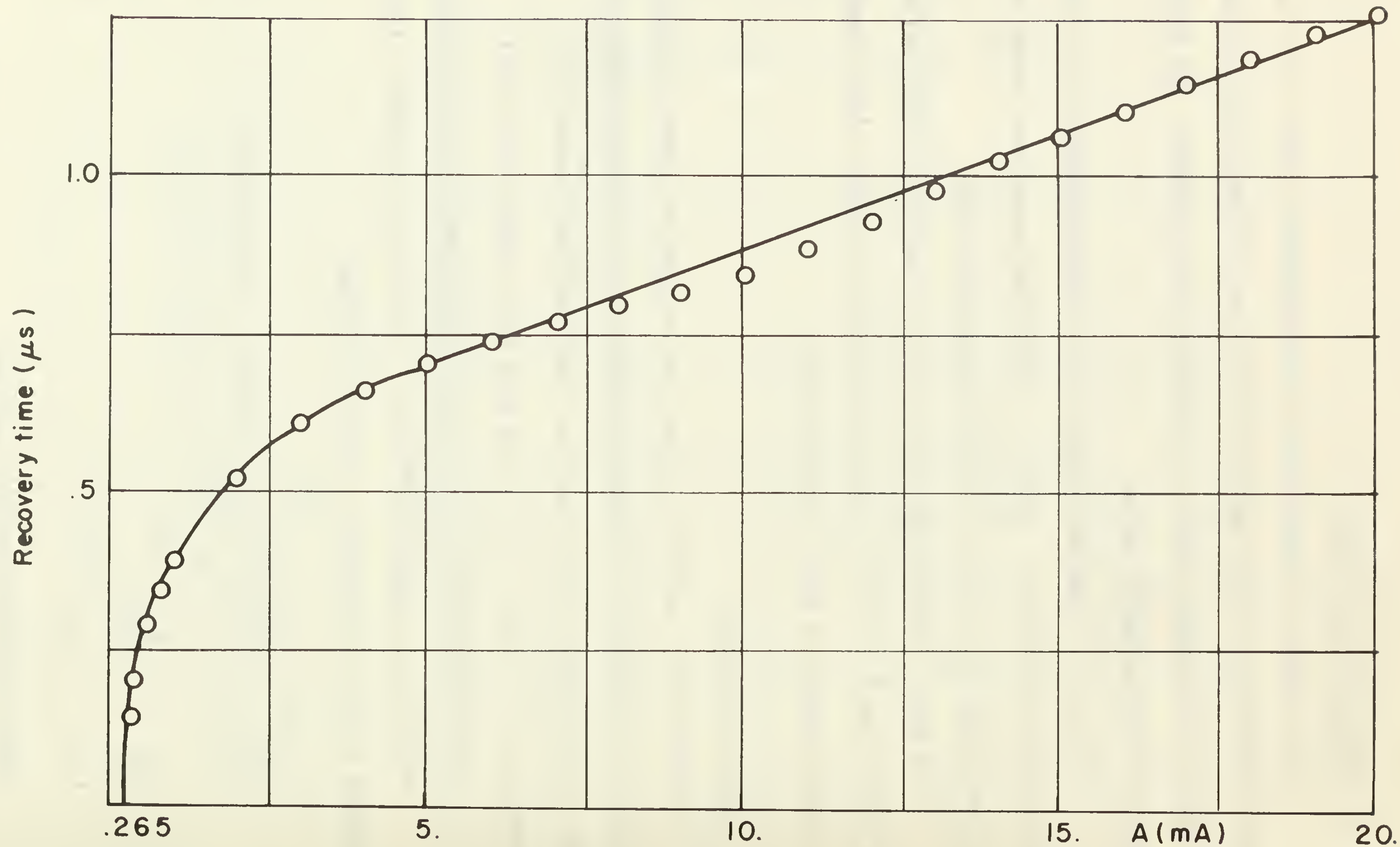


Fig. 3.13. Diode recovery time as a function of radiation current pulse amplitude.

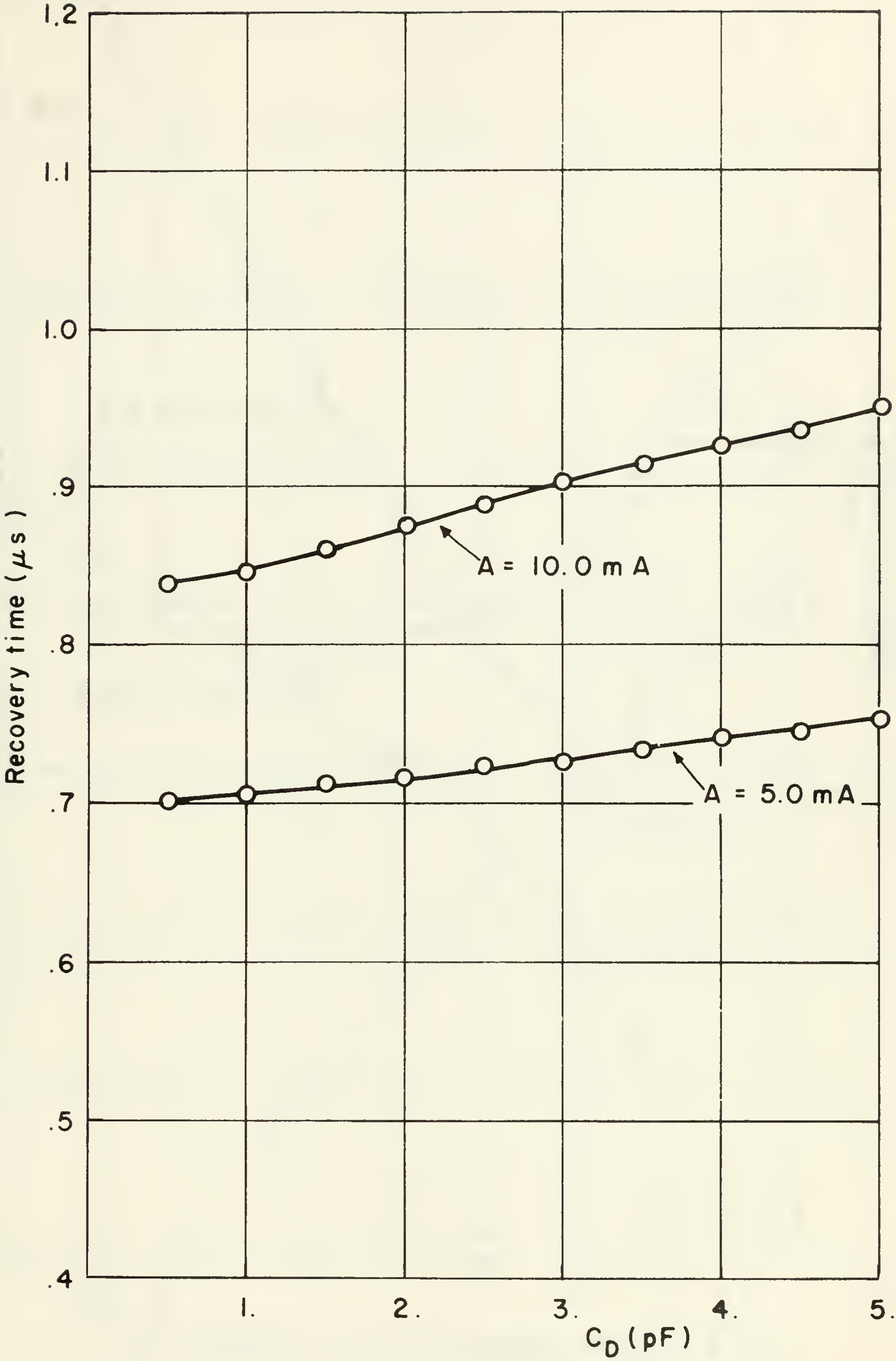


Fig. 3.14. Diode recovery time as a function of C_D .

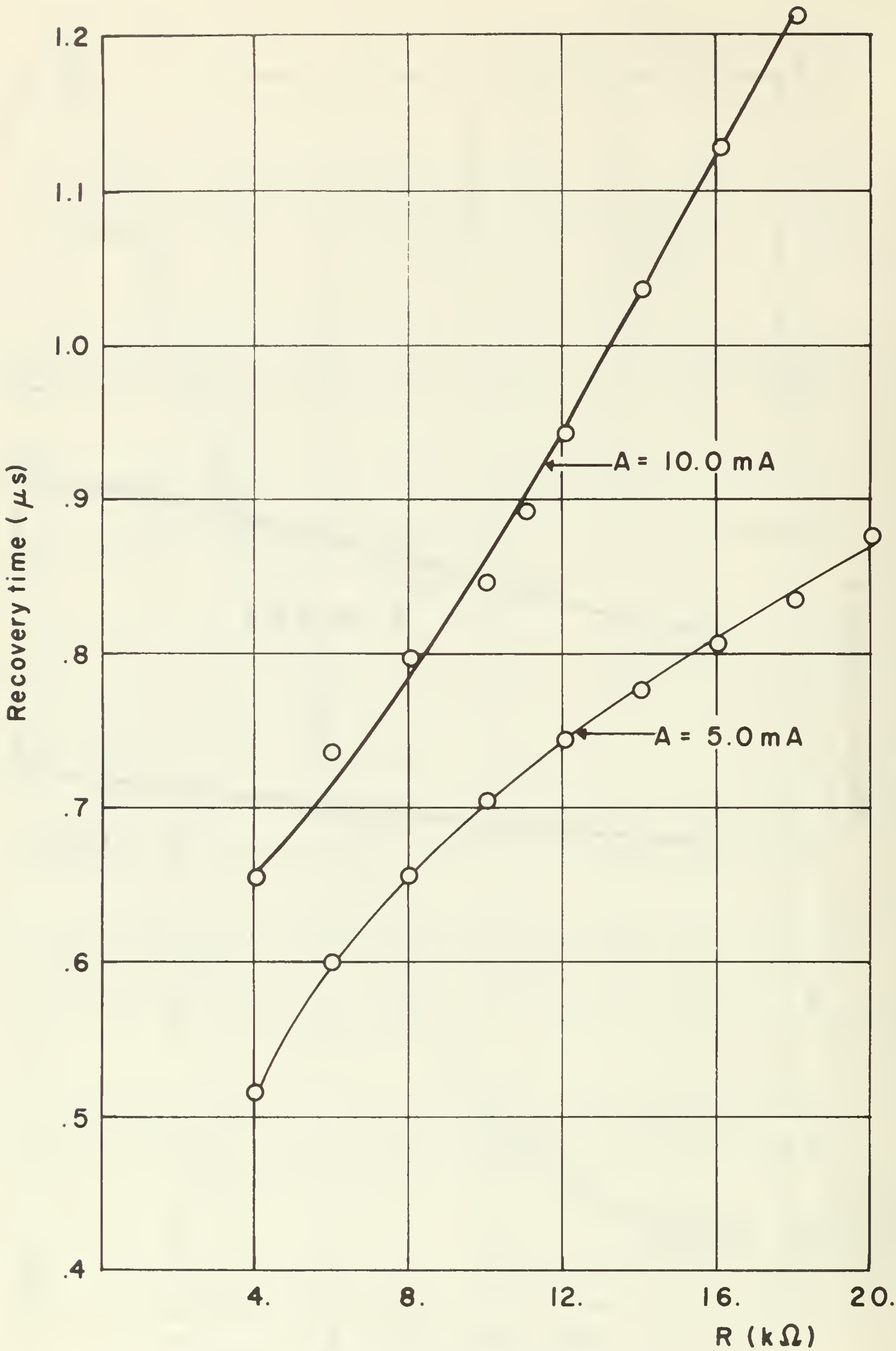


Fig. 3.15. Diode recovery time as a function of R.

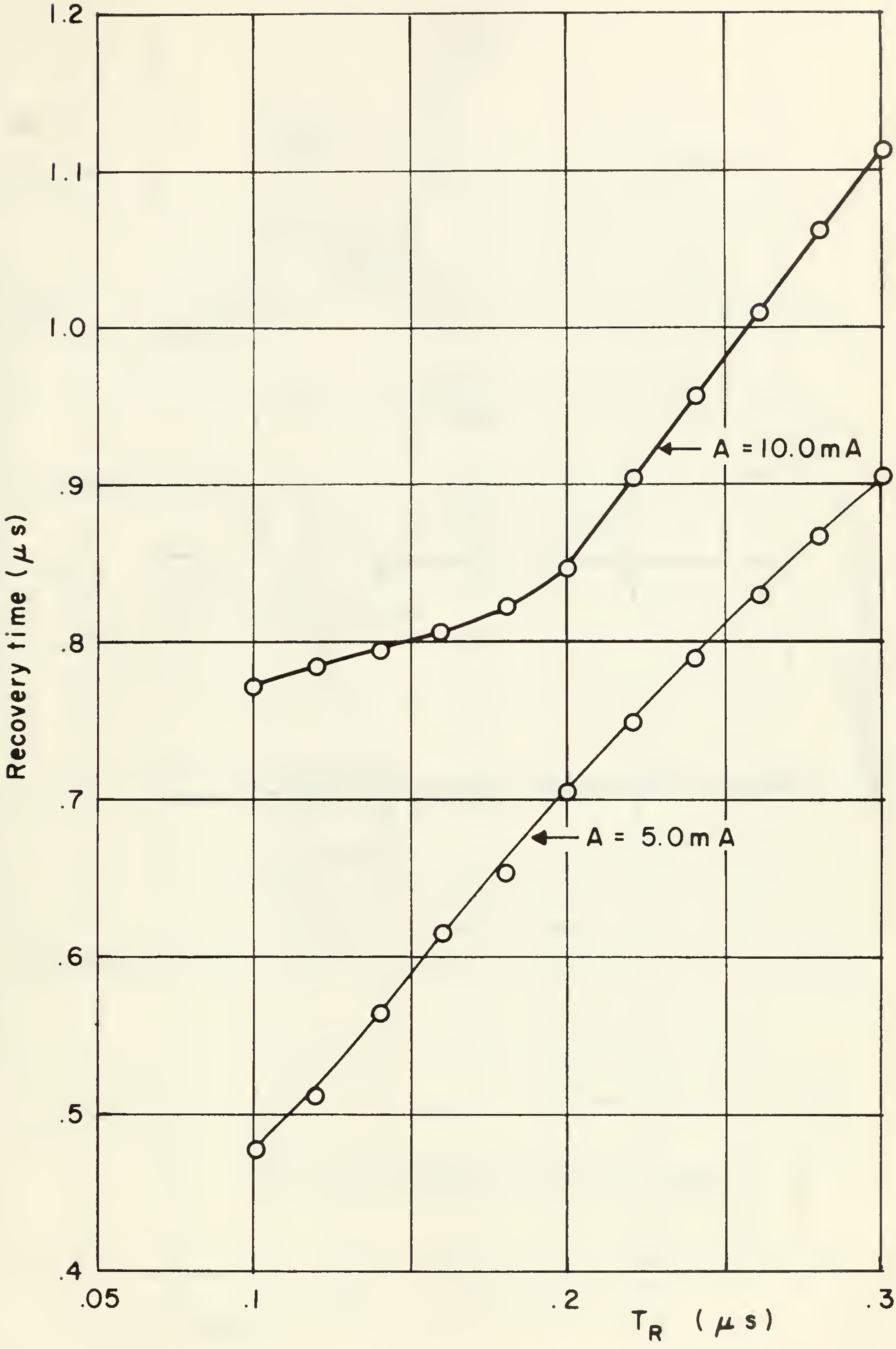


Fig. 3.16. Diode recovery time as a function of T_R .

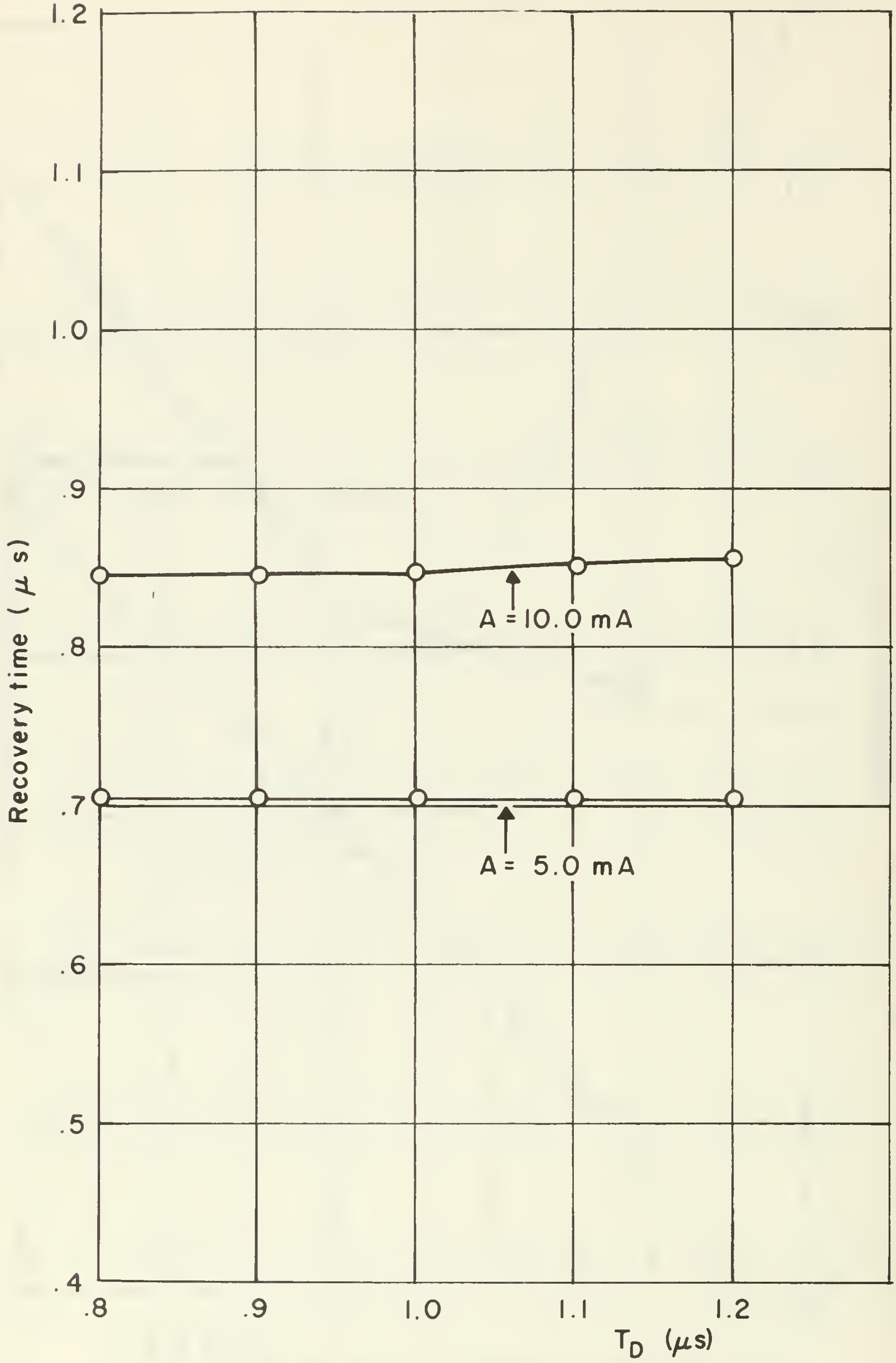


Fig. 3.17. Diode recovery time as a function of T_D .

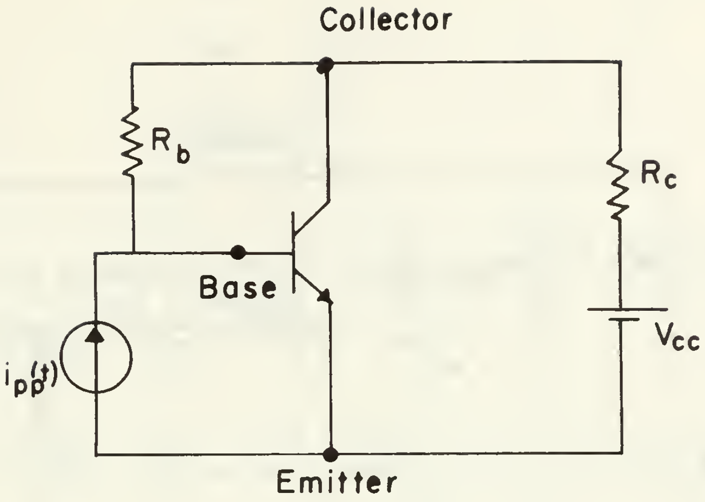


Fig. 3.18. Transistor circuit.

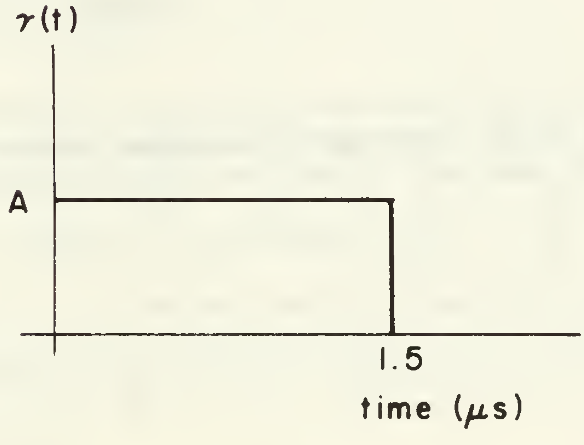


Fig. 3.19. Radiation current pulse.

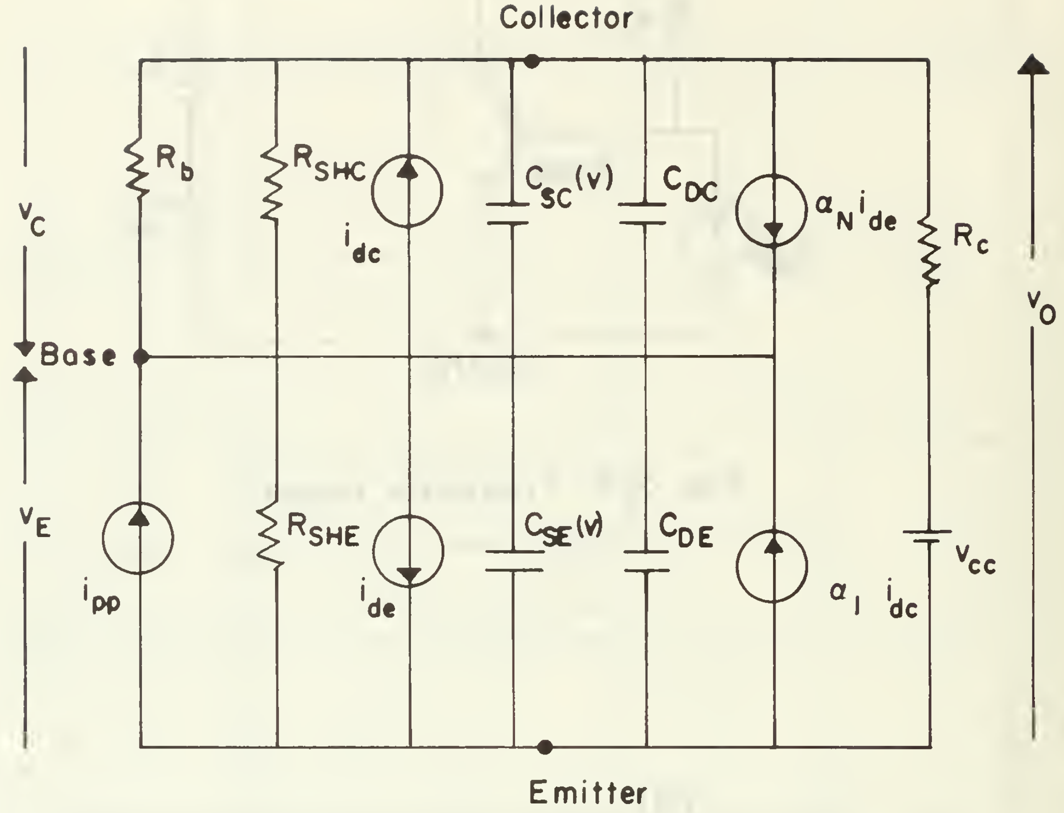


Fig. 3.20. Equivalent transistor circuit.

$\alpha_I = 0.99$ $I_{SE} = 0.1pA$ $T_{DC} = 0.1\mu s$

$T_R = 0.4$ nsec $C_{DE} = 1.0pF$ $\alpha = 38.46\ V^{-1}$

$R_{SHC} = 100\ M\Omega$ $C_{DC} = 1.0pF$

$R_{SHE} = 100\ M\Omega$ $A = 0.1mA$

The circuit equations are obtained by summing the currents at the collector and emitter nodes yielding

$$C_{de}(v)\frac{dv_E}{dt} + \frac{v_E}{R_{SHE}} + i_{de} - \alpha_I i_{dc} - i_{pp} - i_R = 0 \tag{3.37a}$$

$$C_{dc}(v)\frac{dv_C}{dt} + \frac{v_C}{R_{SHC}} + i_{dc} - \alpha_N i_{de} + i_R + \frac{v_C}{R_b} = 0 \tag{3.37b}$$

where

$$i_R = \frac{1}{R_c}(V_{cc} + v_C - v_E) \tag{3.37c}$$

$$C_{dc}(v) = C_{SC}(v) + C_{DC}$$

$$C_{de}(v) = C_{SE}(v) + C_{DE}$$

The biasing resistors, R_b and R_c, are calculated for a desired biasing voltage V_{cc}, base=collector voltage v_C, and base-emitter voltage v_E under steady-state conditions, by setting $\frac{dv_E}{dt}$, $\frac{dv_C}{dt}$ and i_{pp} equal to zero in (3.37). The program used is presented in Appendix B, and the resulting values are $R_b = 0.35\ M\Omega$ and $R_c = 4.5\ k\Omega$ for $V_{cc} = 10.0$ V, $v_E = 0.6$ V and $v_C = -3.4$ V.

Substituting (3.37c) into (3.37a) and (3.37b) and solving for $\dot{\underline{x}} = [\dot{v}_E\ \dot{v}_C]^T$ yields a result identical in form to (3.3) with

$$\underline{f}(\underline{x}(t)) = \begin{bmatrix} -\frac{G_{TE}}{C_{de}(v)} & \frac{1}{C_{de}(v)\cdot R_c} \\ \frac{1}{C_{dc}(v)\cdot R_c} & -\frac{G_{TC}}{C_{dc}(v)} \end{bmatrix} \underline{x}$$

$$+\begin{bmatrix} \left(\dfrac{\alpha_I\, I_{CS}\,(e^{\alpha v_C}-1)}{C_{de}(v)} - \dfrac{I_{ES}(e^{\alpha v_E}-1)}{C_{de}(v)}\right) \\ \\ \left(\dfrac{\alpha_N\, I_{ES}\,(e^{\alpha v_E}-1)}{C_{dc}(v)} - \dfrac{I_{SC}(e^{\alpha v_C}-1)}{C_{dc}(v)}\right) \end{bmatrix} \tag{3.38a}$$

$$B(\underline{x}(t)) = \begin{bmatrix} \dfrac{1}{C_{de}(v)} & \dfrac{1}{C_{de}(v)\, R_c} \\ \\ 0 & -\dfrac{1}{C_{dc}(v)\, R_c} \end{bmatrix} \tag{3.38b}$$

$$\underline{u}(t) = \begin{bmatrix} i_{pp}(t) \\ \\ V_{cc} \end{bmatrix} \tag{3.38c}$$

where $G_{TE} = \dfrac{1}{R_c} + \dfrac{1}{R_{SHE}}$ and $G_{TC} = \dfrac{1}{R_b} + \dfrac{1}{R_c} + \dfrac{1}{R_{SHC}}$.

The iterative solution equation is of the form of (3.9) where $\nabla\underline{f}^T$ and $\nabla(B\underline{u}(n))^T$ are obtained by performing the indicated gradient operation on (3.38).

The computer program for this problem appears in Appendix B, and the results are indicated in Fig. 3.21. The program adjusts ΔT automatically so that the norm, $|\varepsilon|$, never exceeds 0.01 volts.

$$|\varepsilon| = \sqrt{(\Delta v_E)^2 + (\Delta v_C)^2} \tag{3.39}$$

These results have been confirmed by direct integration of the nonlinear state equations using a fourth-order Runge-Kutta integration scheme.

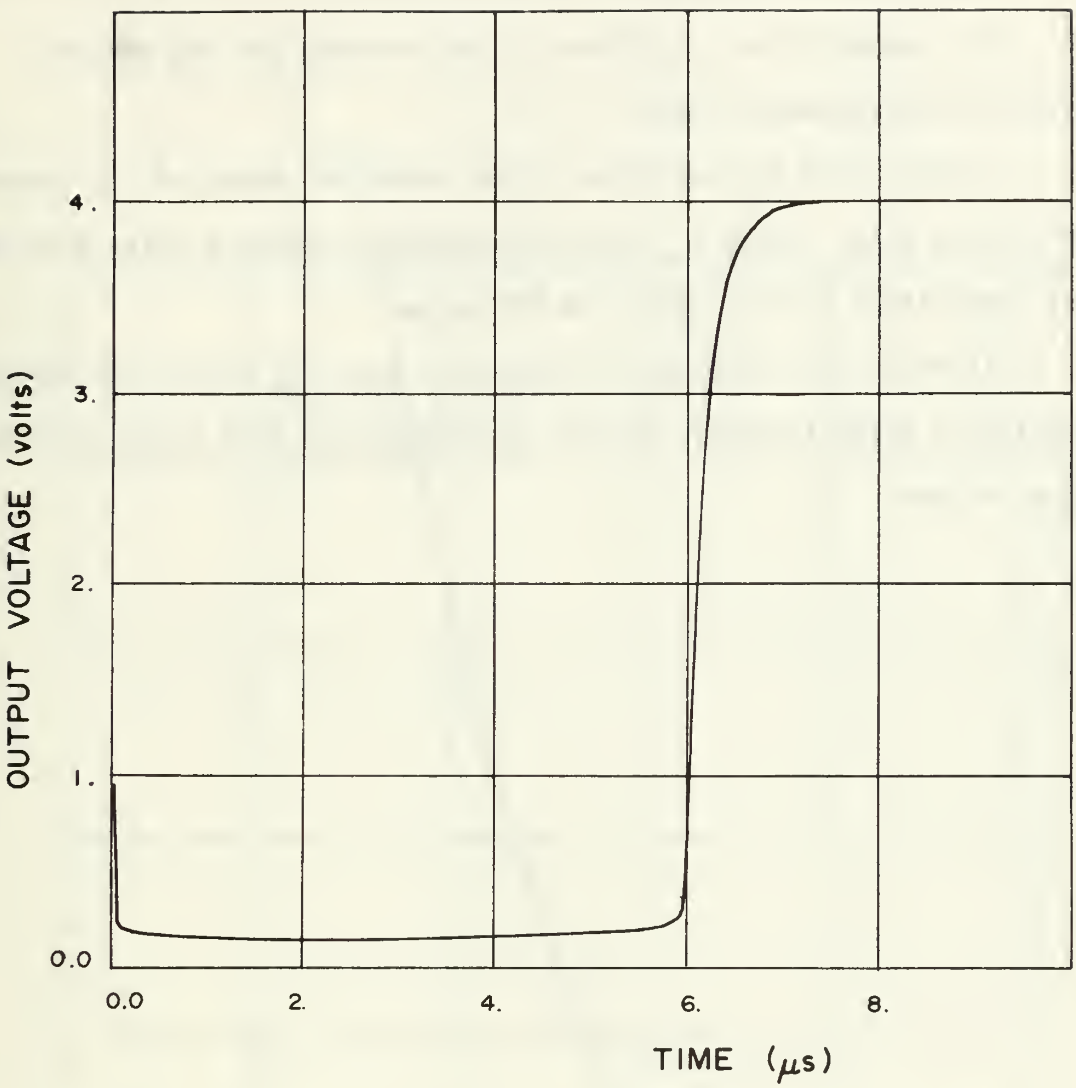


FIG. 3.21. TRANSISTOR RESPONSE TO RADIATION CURRENT PULSE.

2. Presentation of Data

The results of several runs are presented in Figs. 3.22 through 3.25. The recovery time is plotted as the ordinate and the abscissa represents the parameter value.

In Fig. 3.22 the amplitude of the radiation pulse, A, is varied from zero to 3.0mA. When $A \leq 2.0\mu A$ the radiation current pulse does not cause the voltage to drop below the 90% value.

In Figs. 3.23 through 3.25 T_{DC}, T_{DE}, C_{DC}, C_{DE} and T_R are varied respectively while the other circuit parameters are held constant at their nominal values.

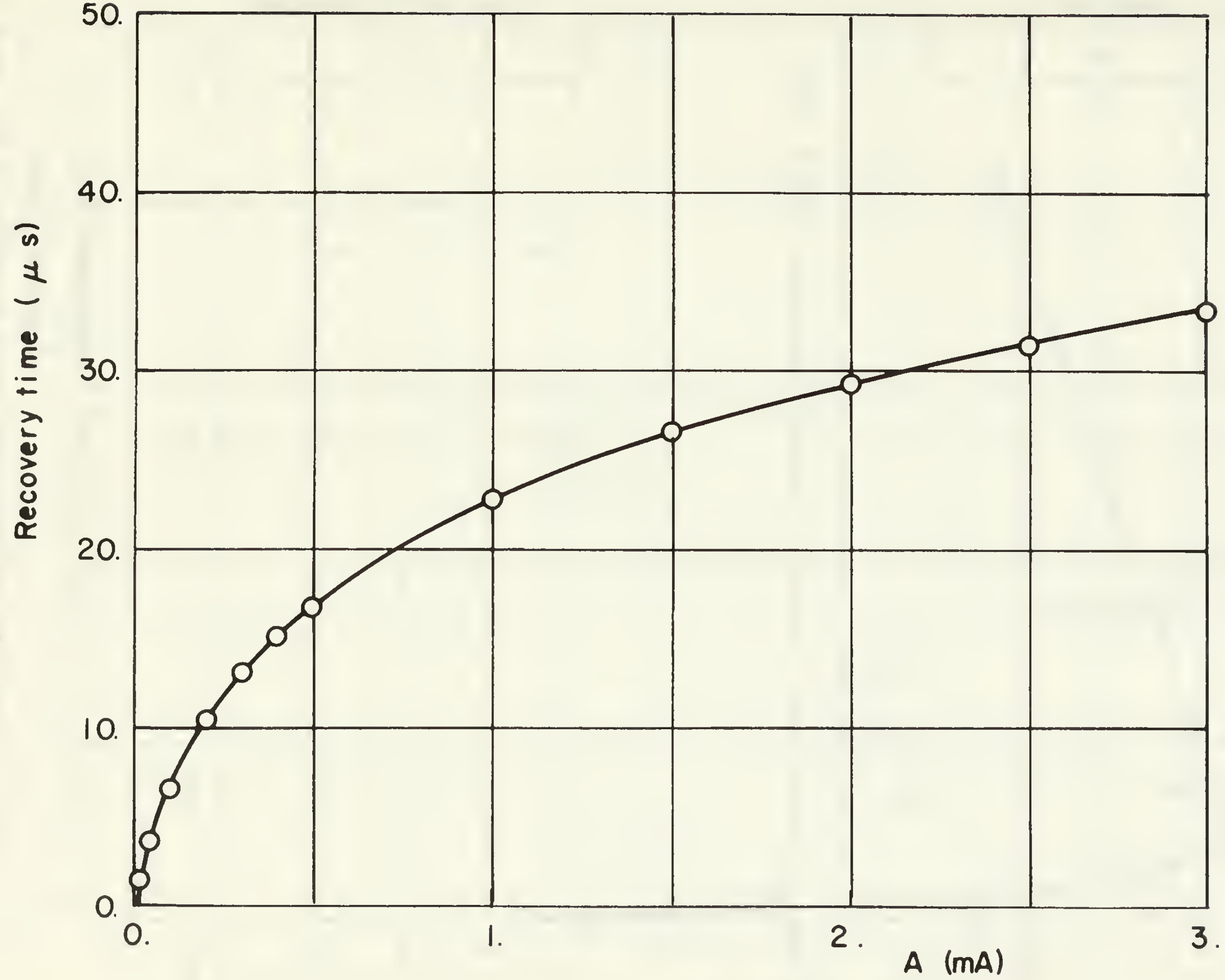


Fig. 3.22. Transistor recovery time as a function of radiation current pulse amplitude.

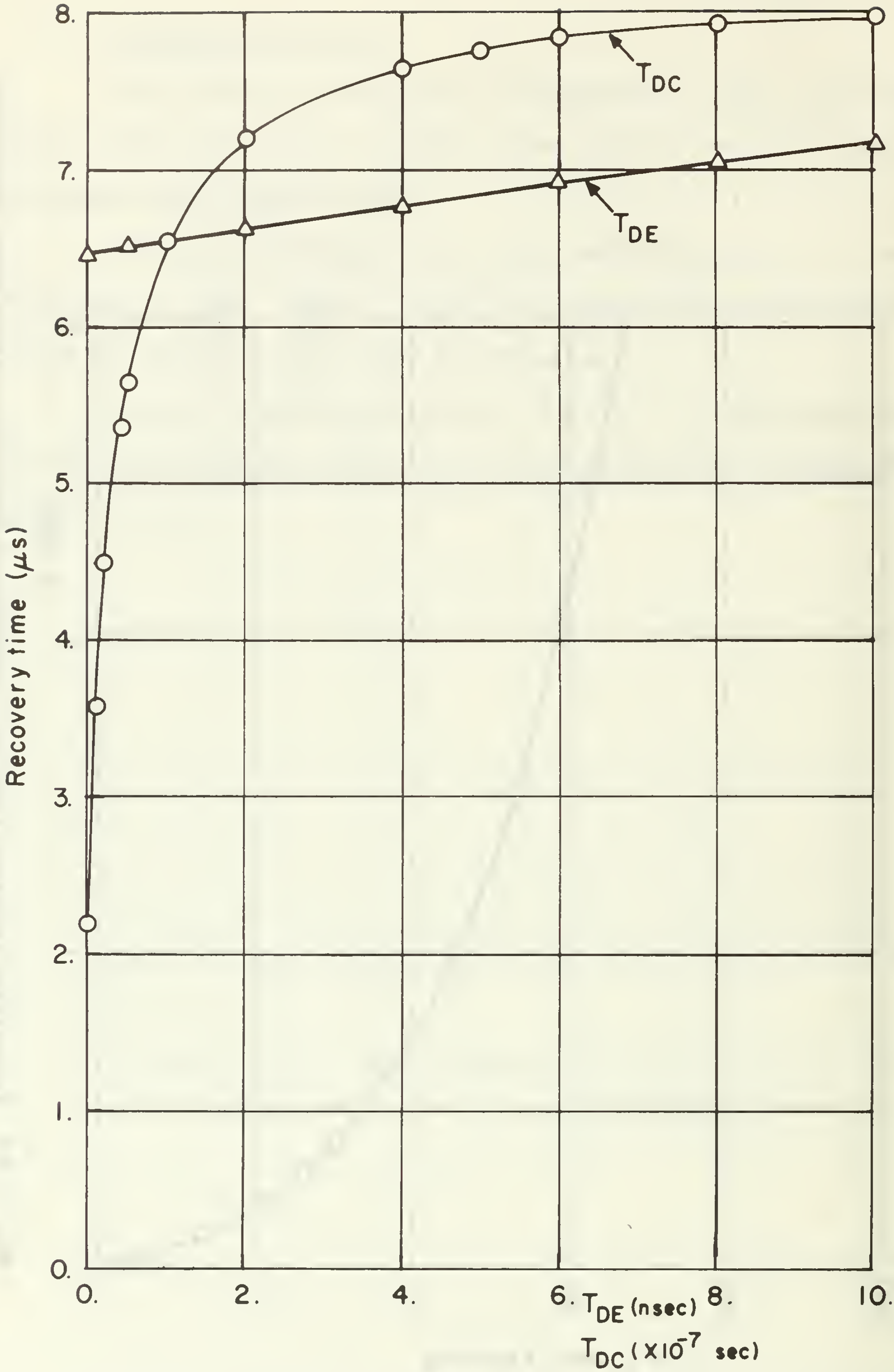


Fig. 3.23. Transistor recovery time as a function of T_{DE} and T_{DC}.

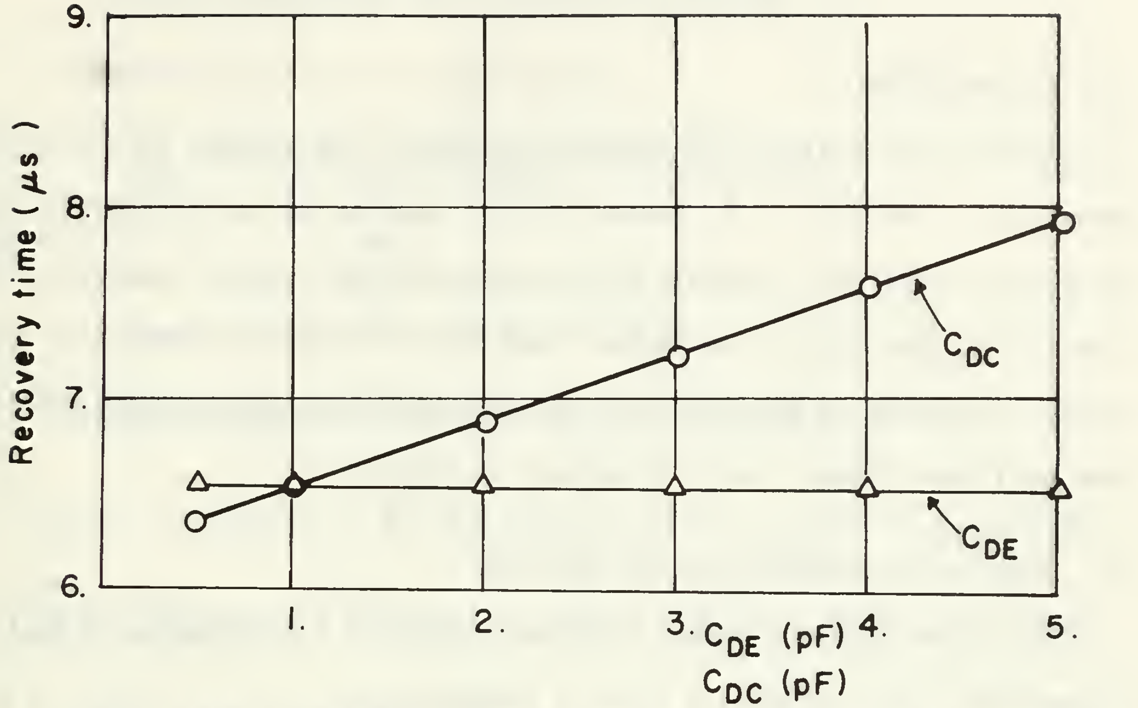


Fig. 3.24. Transistor recovery time as a function of C_{DE} and C_{DC}

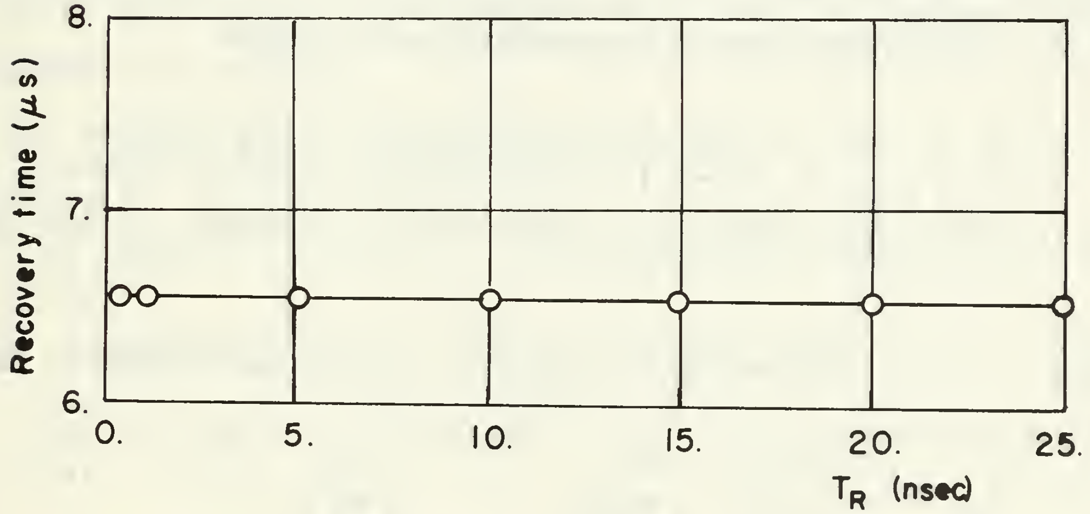


Fig. 3.25. Transistor recovery time as a function of T_R.

IV. SENSITIVITY ANALYSIS OF NONLINEAR CIRCUITS

A. INTRODUCTION

Sensitivity analysis of nonlinear circuits and systems is a relatively new field. Recently S. R. Parker [4] has considered the sensitivity analysis of general nonlinear circuits utilizing auxiliary coupled networks. The sensitivity analysis of nonlinear circuits utilizing the concept of state space is covered in the next section, followed by a sensitivity analysis of the nonlinear diode circuit discussed in Chapter III.

B. SENSITIVITY-AUGMENTED STATE EQUATION

The state equation format adopted in Chapter III (repeated below) is convenient when considering circuit sensitivity.

$$\dot{\underline{x}}(t) = A(\underline{x}(t))\cdot\underline{x}(t) + \underline{c}(\underline{x}(t)) + B(\underline{x}(t))\cdot\underline{u}(t) \tag{4.1}$$

As in the linear case, the sensitivity function, $S_{a_j}^{x_i} = \dfrac{\partial x_i}{\partial \ln a_j}$, may be obtained by performing the indicated partial differentiation on (4.1), yielding the sensitivity-augmented state equation.

$$\begin{bmatrix} \dot{\underline{x}} \\ \hline \dot{\underline{x}}_{sj} \end{bmatrix} = \left[\begin{array}{c|c} A & 0 \\ \hline 0 & A_D \end{array}\right] \begin{bmatrix} \underline{x} \\ \hline \underline{x}_{sj} \end{bmatrix} + \begin{bmatrix} \underline{c} \\ \hline \underline{c}_{sj} \end{bmatrix} + \left[\begin{array}{c|c} B & 0 \\ \hline 0 & B_D \end{array}\right] \begin{bmatrix} \underline{u} \\ \hline \underline{u}_{sj} \end{bmatrix} + \left[\begin{array}{c|c} 0 & 0 \\ \hline A_{sj} & B_{sj} \end{array}\right] \begin{bmatrix} \underline{x} \\ \hline \underline{u} \end{bmatrix} \tag{4.2}$$

where $j = 1,2, \ldots, r$ and $\underline{x}_{sj}$, A_D, B_D, A_{sj}, B_{sj}, $\underline{u}_{sj}$ are defined in Chapter II and

$$\underline{c}_{sj} = \begin{bmatrix} \dfrac{\partial c_1}{\partial \ln a_j} & \dfrac{\partial c_2}{\partial \ln a_j} & \cdots & \dfrac{\partial c_n}{\partial \ln a_j} \end{bmatrix}^T \tag{4.3}$$

Separating the set of first-order partial differential equations for each sensitivity parameter in (4.2) yields

$$\dot{\underline{x}}_{sj} = A\,\underline{x}_{sj} + \underline{c}_{sj} + A_{sj}\,\underline{x} + B\,\underline{u}_{sj} + B_{sj}\,\underline{u} \tag{4.4}$$

for $j = 1,2, \ldots, r$

Equation (4.4) may be rewritten as

$$\dot{\underline{x}}_{sj} = \underline{f}_{sj}(\underline{x},\underline{x}_{sj}) + B(\underline{x})\underline{u}_{sj} + B_{sj}(\underline{x},\underline{x}_{sj})\underline{u} \tag{4.5}$$

where $j = 1,2, \ldots ,r$ and

$$\underline{f}_{sj}(\underline{x},\underline{x}_{sj}) = A\,\underline{x}_{sj} + \underline{c}_{sj} + A_{sj}\,\underline{x}$$

Applying trapezoidal integration to (4.5) yields

$$\underline{x}_{sj}(n) - \underline{x}_{sj}(n-1) = \frac{\Delta T}{2}\,[\underline{f}_{sj}(\underline{x}(n),\underline{x}_{sj}(n)) + \underline{f}_{sj}(\underline{x}(n-1),\underline{x}_{sj}(n-1))]$$

$$+ \frac{\Delta T}{2}\,[B(\underline{x}(n))\underline{u}_{sj}(n) + B(\underline{x}(n-1))\underline{u}_{sj}(n-1)] + \frac{\Delta T}{2}\,[B_{sj}(\underline{x}(n),\underline{x}_{sj}(n))\;\underline{u}(n)$$

$$+ B_{sj}(\underline{x}(n-1),\underline{x}_{sj}(n-1))\;\underline{u}(n-1)] \tag{4.6}$$

The functions $\underline{f}_{sj}(\underline{x}(n),\underline{x}_{sj}(n))$ and $B_{sj}(\underline{x}(n),\underline{x}_{sj}(n))\underline{u}(n)$, where $\underline{u}(n)$ is considered a constant, can be expanded in a Taylor series about the point $(\underline{x}(n-1),\underline{x}_{sj}(n-1))$ and approximated by the first two terms, as in Chapter III, yielding

$$\underline{f}_{sj}(\underline{x}(n),\underline{x}_{sj}(n)) = \underline{f}_{sj}(\underline{x}(n-1),\underline{x}_{sj}(n-1))$$

$$+ (\underline{\Delta x}^T\nabla_x)\underline{f}_{sj}(\underline{x}(n-1),\underline{x}_{sj}(n-1)) + (\underline{\Delta x}_{sj}^T\nabla_{xsj})\underline{f}_{sj}(\underline{x}(n-1),\underline{x}_{sj}(n-1)) \tag{4.7a}$$

$$B_{sj}(\underline{x}(n),\underline{x}_{sj}(n))\underline{u}(n) = B_{sj}(\underline{x}(n-1),\underline{x}_{sj}(n-1))\underline{u}(n)$$

$$+ (\underline{\Delta x}^T\nabla_x)[B_{sj}(\underline{x}(n-1),\underline{x}_{sj}(n-1))\underline{u}(n)] + (\underline{\Delta x}_{sj}^T\nabla_{xsj})[B_{sj}(\underline{x}(n-1),x_{sj}(n-1))\;\underline{u}(n)] \tag{4.7b}$$

where

$$\nabla_x = \left[\frac{\partial}{\partial x_1} \quad \cdots \quad \frac{\partial}{\partial x_n}\right]^T$$

$$\nabla_{xsj} = \left[\frac{\partial}{\partial {x_1}_{sj}} \quad \cdots \quad \frac{\partial}{\partial {x_n}_{sj}}\right]^T$$

$B(\underline{x}(n))\cdot \underline{u}_{sj}(n)$, where $\underline{u}_{sj}(n)$ is considered constant, can be expanded in a Taylor series about the point $\underline{x}(n-1)$ and approximated by the first two terms yielding:

$$B(\underline{x}(n))\cdot \underline{u}_{sj}(n) = B(\underline{x}(n-1))\cdot \underline{u}_{sj}(n) + (\underline{\Delta x}^T \nabla_x)[B(\underline{x}(n-1))\cdot \underline{u}_{sj}(n)] \tag{4.7c}$$

Substituting (4.7) into (4.6) yields

$$\underline{x}_{sj}(n) - \underline{x}_{sj}(n-1) = \frac{\Delta T}{2}[2\underline{f}_{sj} + (\underline{\Delta x}^T \nabla_x)\underline{f}_{sj} + (\underline{\Delta x}_{sj}^T \nabla_{xsj})\underline{f}_{sj}]$$

$$+ \frac{\Delta T}{2}[B\cdot(\underline{u}_{sj}(n) + \underline{u}_{sj}(n-1)) + (\underline{\Delta x}^T \nabla_x)(B\underline{u}_{sj}(n))]$$

$$+ \frac{\Delta T}{2}[B_{sj}\cdot(\underline{u}(n) + \underline{u}(n-1)) + (\underline{\Delta x}^T \nabla_x)(B_{sj}\underline{u}(n)) + (\underline{\Delta x}_{sj}^T \nabla_{xsj})(B_{sj}\underline{u}(n))] \tag{4.8}$$

where $\underline{f}_{sj}$, B_{sj} and $B_{sj}\underline{u}(n)$ are evaluated at the point $(\underline{x}(n-1))$,

Recall that

$$(\underline{\Delta x}_{sj}^T \nabla_{xsj})\,\underline{f}_{sj} = (\nabla_{xsj}\,\underline{f}_{sj}^T)^T\,\underline{\Delta x}_{sj} \tag{4.9}$$

Substituting (4.9) into (4.8) and solving for $\underline{x}_{sj}(n)$ yields

$$\underline{x}_{sj}(n) = \underline{x}_{sj}(n-1) + \Delta T\,[I - \frac{\Delta T}{2}\{(\nabla_{xsj}\underline{f}_{sj}^T)^T + (\nabla_{xsj}[B_{sj}\underline{u}(n)]^T)^T\}\;]^{-1}$$

$$[\underline{f}_{sj} + \frac{1}{2}\{(\underline{\Delta x}^T \nabla_x)\underline{f}_{sj} + B\cdot(\underline{u}_{sj}(n) + \underline{u}_{sj}(n-1)) + (\underline{\Delta x}^T \nabla_x)[B\underline{u}_{sj}(n)]$$

$$+ B_{sj}\cdot(\underline{u}(n) + \underline{u}(n-1)) + (\underline{\Delta x}^T \nabla_x)[B_{sj}\underline{u}(n)]\}\;] \tag{4.10}$$

Equations (3.9) and (4.10) can be solved directly for $\underline{x}(n)$ and $\underline{x}_{sj}(n)$ from the known values of $\underline{x}(n-1)$, $\underline{u}(n)$, $\underline{u}(n-1)$, $\underline{u}_{sj}(n)$, and $\underline{u}_{sj}(n-1)$. The solution yields the time response of the states and their corresponding sensitivities.

C. NONLINEAR EXAMPLE

In order to illustrate the technique discussed above, sensitivity analysis of the diode circuit in Chapter III is considered. The state equation for the diode circuit of Fig. 3.5 contains a single state variable. Equation (4.10) reduces to

$$v_{sj}(n) = v_{sj}(n-1) + \Delta T\cdot[f_{sj} + \frac{1}{2}\{\frac{\partial f_{sj}}{\partial v}\Delta v + B\cdot(u_{sj}(n) + u_{sj}(n-1)) + \frac{\partial(B\cdot u_{sj}(n))}{\partial v}\Delta v + B_{sj}\cdot(u(n) + u(n-1)) + \frac{\partial(B_{sj}u(n))}{\partial v}\Delta v\}]$$

$$[1 - \frac{\Delta T}{2}\{\frac{\partial f_{sj}}{\partial v_{sj}} + \frac{\partial(B_{sj}u(n))}{\partial v_{sj}}\}]^{-1} \tag{4.11}$$

where f_{sj}, $\frac{\partial f_{sj}}{\partial v}$, B, B_{sj}, $\frac{\partial(B\cdot u_{sj}(n))}{\partial v}$ and $\frac{\partial(B_{sj}u(n))}{\partial v}$ are evaluated at the point $v(n-1)$.

The state equation may be written in the form of (4.1) where

$$A(v(t)) = -\frac{G}{C_D + T_D I_S \alpha e^{\alpha v}} \tag{4.12a}$$

$$c(v(t)) = -\frac{I_S(e^{\alpha v} - 1)}{C_D + T_D I_S \alpha e^{\alpha v}} \tag{4.12b}$$

and $B(v(t))$ and $u(t)$ are given in (3.34b) and (3.34c) respectively.

The equations for the sensitivity functions take the form of (4.5). The sensitivity parameters considered are the diode parameters, T_D and C_D, the radiation-pulse parameters, A and T_R, and the circuit parameters, R and E. The terms in (4.5) are presented in table 4.1 for each of the sensitivity parameters where

$$k_1 = I_S \alpha e^{\alpha v} \qquad\qquad k_2 = T_D k_1$$

Sensitivity Function	$f_{sj}(v,v_{sj})$	$B_{sj}(v,v_{sj})$	u_{sj}
$v_{s1} = \dfrac{\partial v}{\partial \ln T_D}$	$-\dfrac{(G + k_1)\ v_{s1}}{D} + \dfrac{(k_2 + k_3 v_{s1})(Gv + k_4)}{D^2}$	$-\dfrac{(k_2 + k_3 v_{s1})}{D^2}$	0
$v_{s2} = \dfrac{\partial v}{\partial \ln E}$	$-\dfrac{(G + k_1)\ v_{s2}}{D} + \dfrac{k_3 v_{s2}(Gv + k_4)}{D^2}$	$-\dfrac{k_3 v_{s2}}{D^2}$	$-E/R$
$v_{s3} = \dfrac{\partial v}{\partial \ln R}$	$-\dfrac{(G + k_1)v_{s3} + (v/R)}{D} + \dfrac{k_3 v_{s3}(Gv + k_4)}{D^2}$	$-\dfrac{k_3 v_{s3}}{D^2}$	E/R
$v_{s4} = \dfrac{\partial v}{\partial \ln A}$	$-\dfrac{(G + k_1)v_{s5}}{D} + \dfrac{k_3 v_{s4}(Gv + k_4)}{D^2}$	$-\dfrac{k_3 v_{s4}}{D^2}$	$\dfrac{\partial\, i_{pp}}{\partial \ln A}$
$v_{s5} = \dfrac{\partial v}{\partial \ln C_D}$	$-\dfrac{(G + k_1)v_{s5}}{D} + \dfrac{(C_D + k_3 v_{s5})(Gv + k_4)}{D^2}$	$-\dfrac{C_D + k_3 v_{s5}}{D^2}$	0
$v_{s6} = \dfrac{\partial v}{\partial \ln T_R}$	$-\dfrac{(G + k_1)v_{s6}}{D} + \dfrac{k_3 v_{s6}(Gv + k_4)}{D^2}$	$-\dfrac{k_3 v_{s6}}{D^2}$	$\dfrac{\partial\, i_{pp}}{\partial \ln T_R}$

Table 4.1. Terms of (4.5) for the circuit of Fig. 3.6.

$$k_3 = \alpha k_2 \qquad\qquad k_4 = I_S(e^{\alpha v} - 1)$$

$$D = C_D + k_2$$

The expressions for $\frac{\partial i_{pp}}{\partial \ln A}$ and $\frac{\partial i_{pp}}{\partial \ln T_R}$ are obtained by performing the indicated partial differentiation on (3.21) and $\frac{\partial f_{sj}}{\partial v}$, $\frac{\partial(B \cdot u_{sj}(n))}{\partial v}$, $\frac{\partial(B_{sj}u(n))}{\partial v}$, $\frac{\partial f_{sj}}{\partial v_{sj}}$, and $\frac{\partial(B_{sj}u(n))}{\partial v_{sj}}$ are obtained by performing the indicated partial differentiation on the equations listed in table 4.1.

In addition to the sensitivity functions the voltage response, $v_o = -v$, is shown for (1) nominal parameter values, and (2) a 10% change in the sensitivity parameter value. The response is presented in Figs. 4.2, 4.4, 4.6, 4.8, 4.10, and 4.12. The computer program for generating the sensitivity functions is given in Appendix B, and the resulting sensitivity functions are presented in Figs. 4.1, 4.3, 4.5, 4.7, 4.9, and 4.11. The program adjusts ΔT automatically so that $|\varepsilon_j|$ never exceeds 0.05 volts and $|\Delta v|$ never exceeds 0.01 volts, where

$$|\varepsilon_j| = \sqrt{(\Delta v)^2 + (\Delta v_{sj})^2} \qquad \text{for } j = 1, 2, \ldots, 6 \tag{4.13}$$

These results have been confirmed by direct integration of the non-linear state equations using a fourth-order Runge-Kutta integration scheme. It should be noted that the method outlined in this chapter is approximately twice as fast as the fourth-order Runge-Kutta integration scheme.

D. INTERPRETATION AND APPLICATION OF THE SENSITIVITY FUNCTION

The definition of the sensitivity function adopted in this thesis is $S_{a_j}^{x_i} = \frac{\partial x_i}{\partial \ln a_j} = \frac{\partial x_i}{(\partial a_j/a_j)}$. As noted, this definition applies only for infinitesimally small changes in x_i and a_j. The sensitivity function may

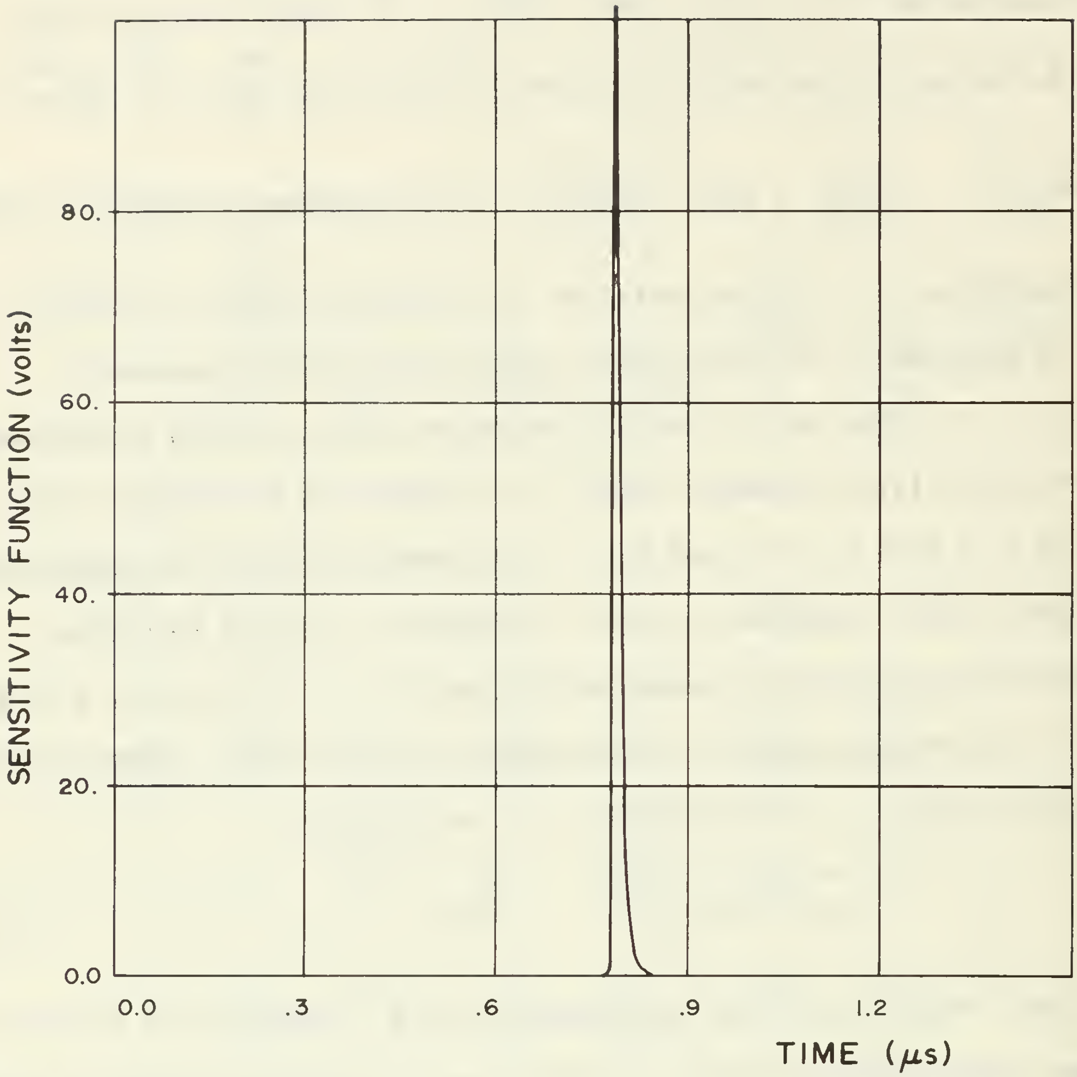


FIG. 4.1. SENSITIVITY FUNCTION WITH RESPECT TO T_D.

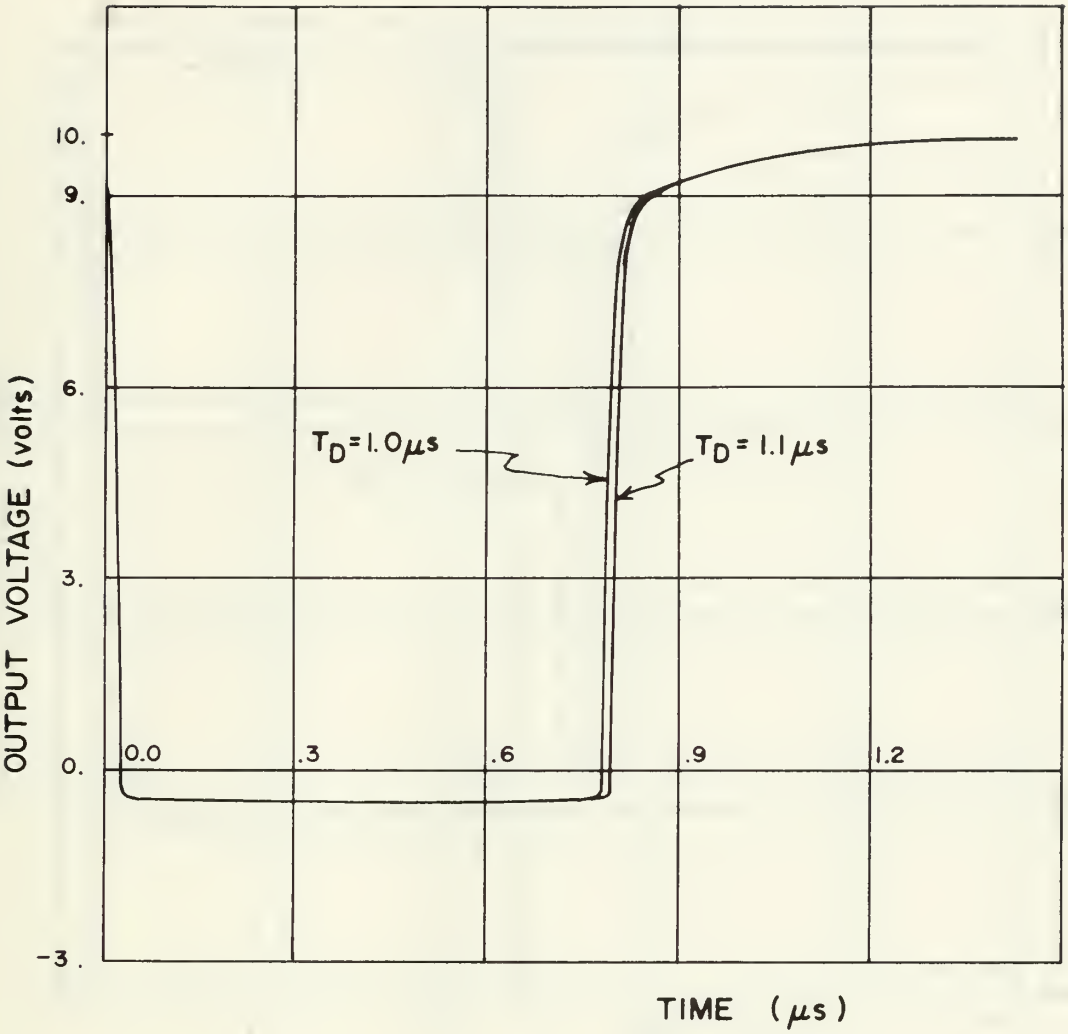


FIG. 4.2. RESPONSE OF DIODE CIRCUIT TO RADIATION PULSE.

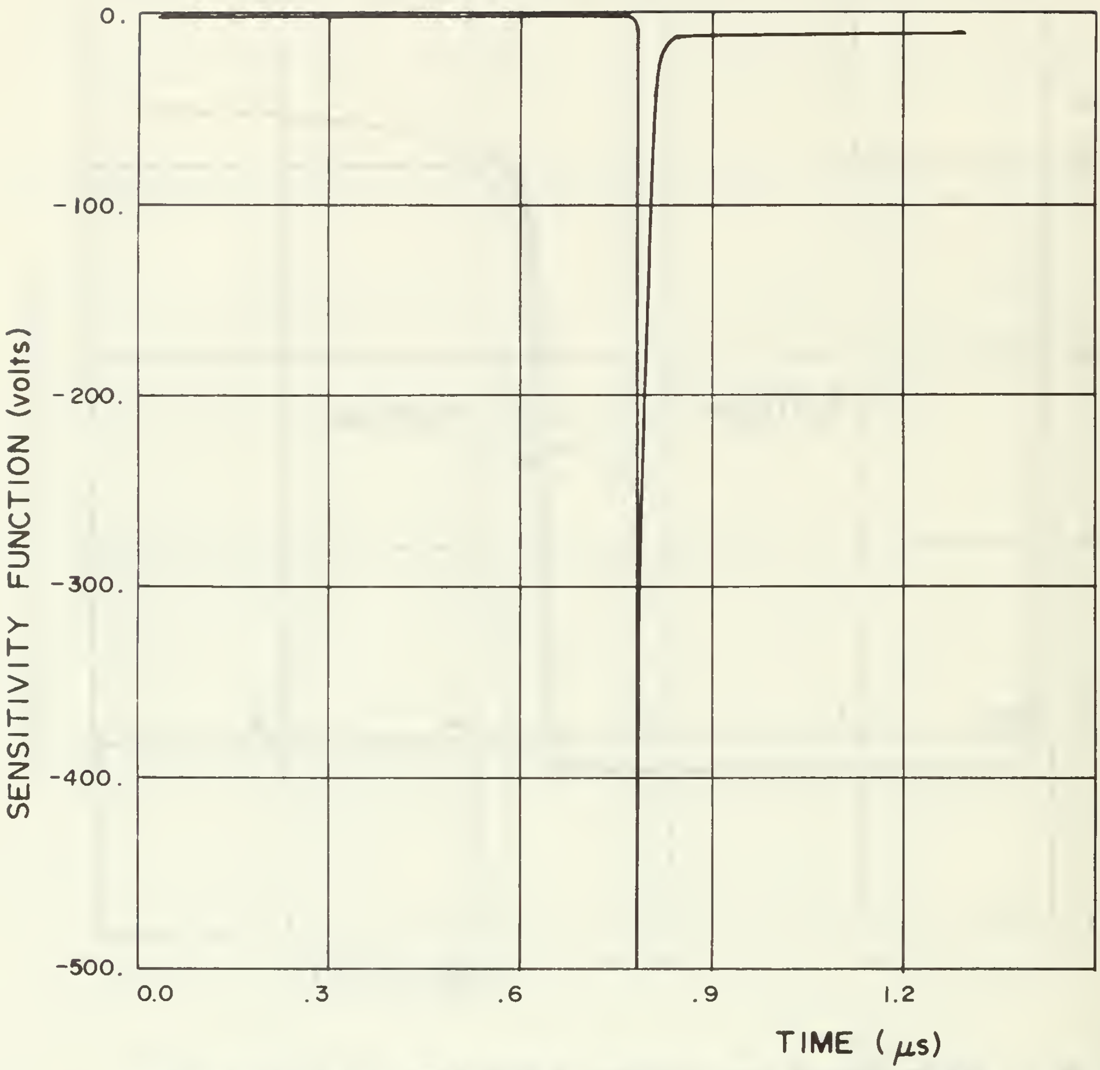


FIG. 4.3. SENSITIVITY FUNCTION WITH RESPECT TO E.

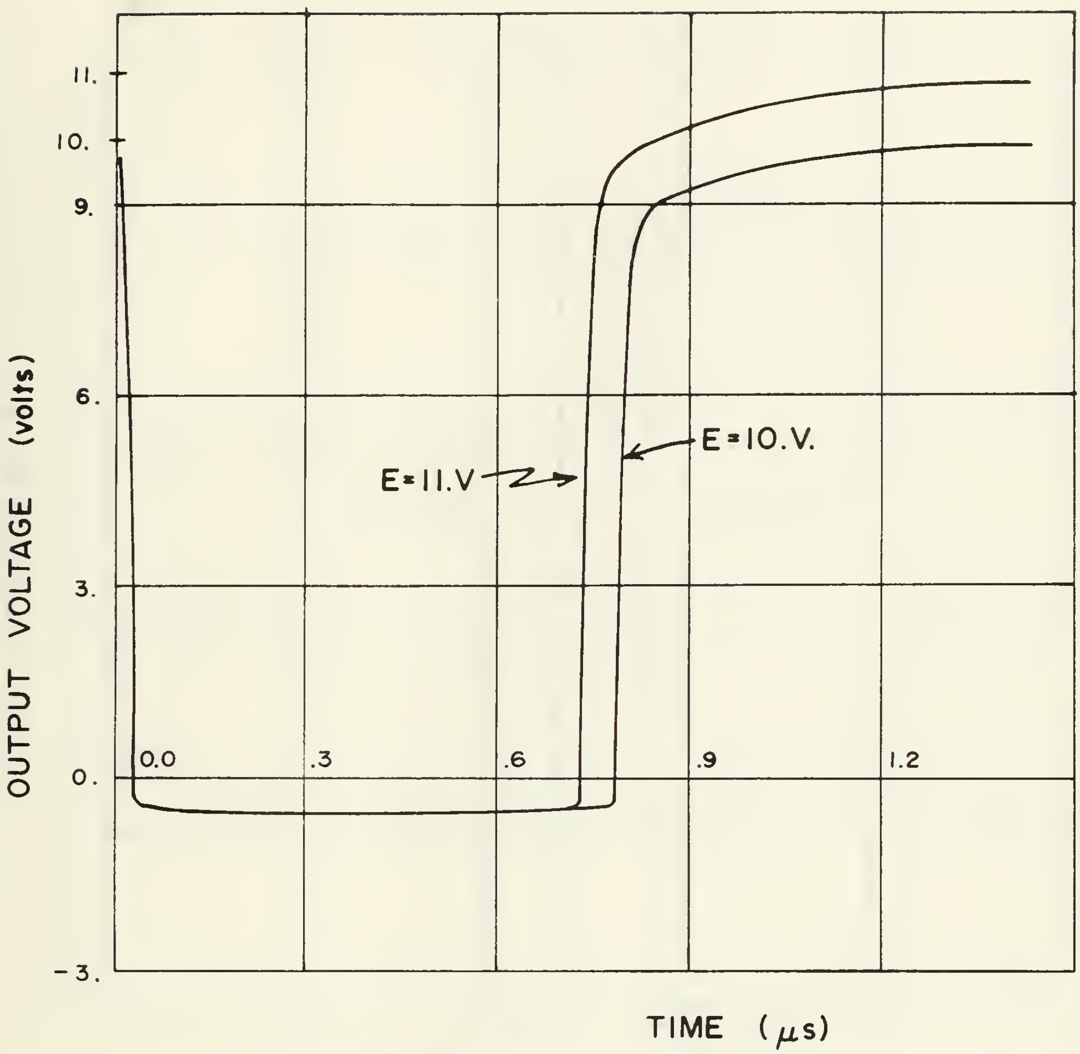


FIG. 4.4. RESPONSE OF DIODE CIRCUIT TO RADIATION PULSE.

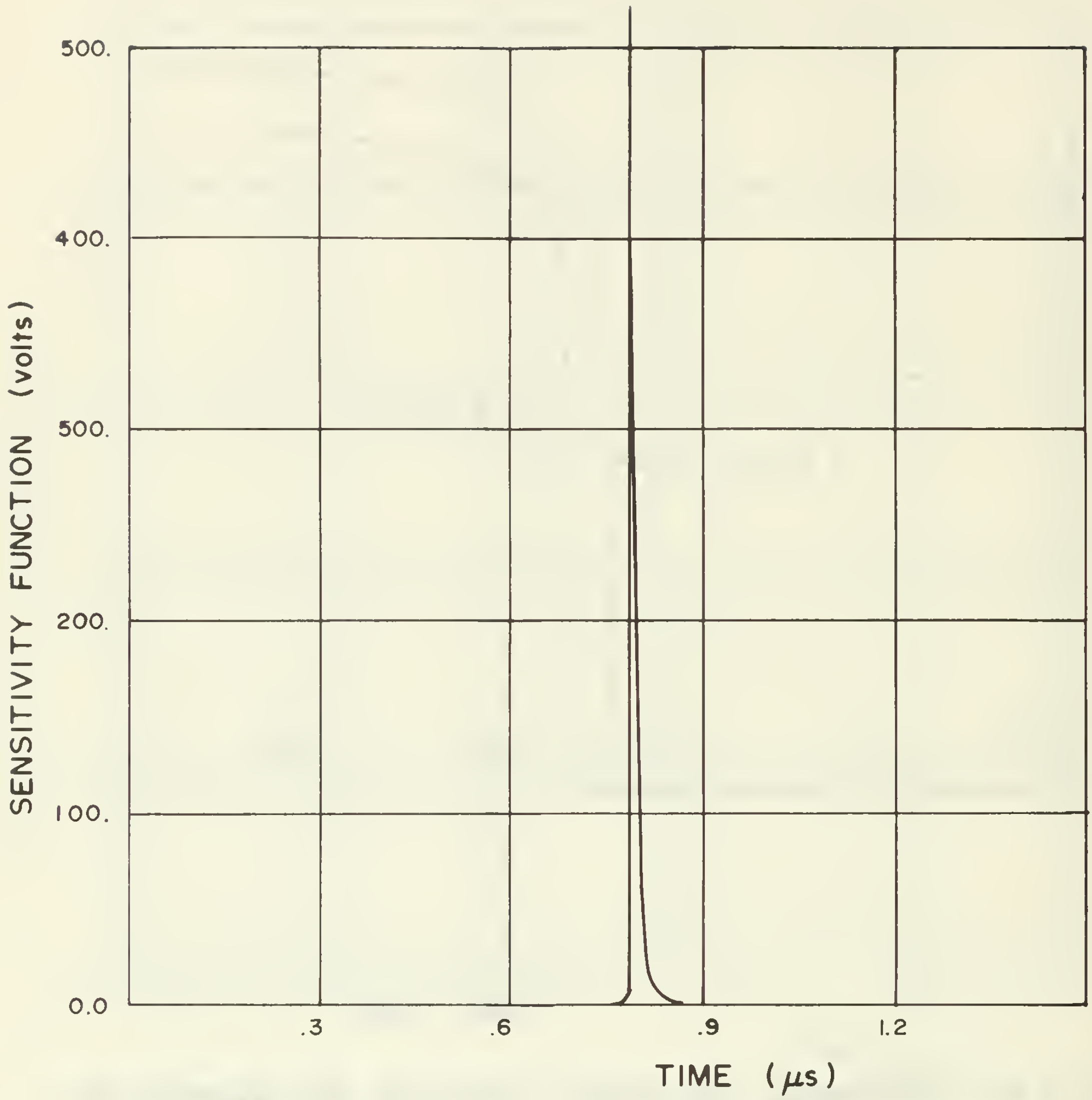


FIG. 4.5. SENSITIVITY FUNCTION WITH RESPECT TO R.

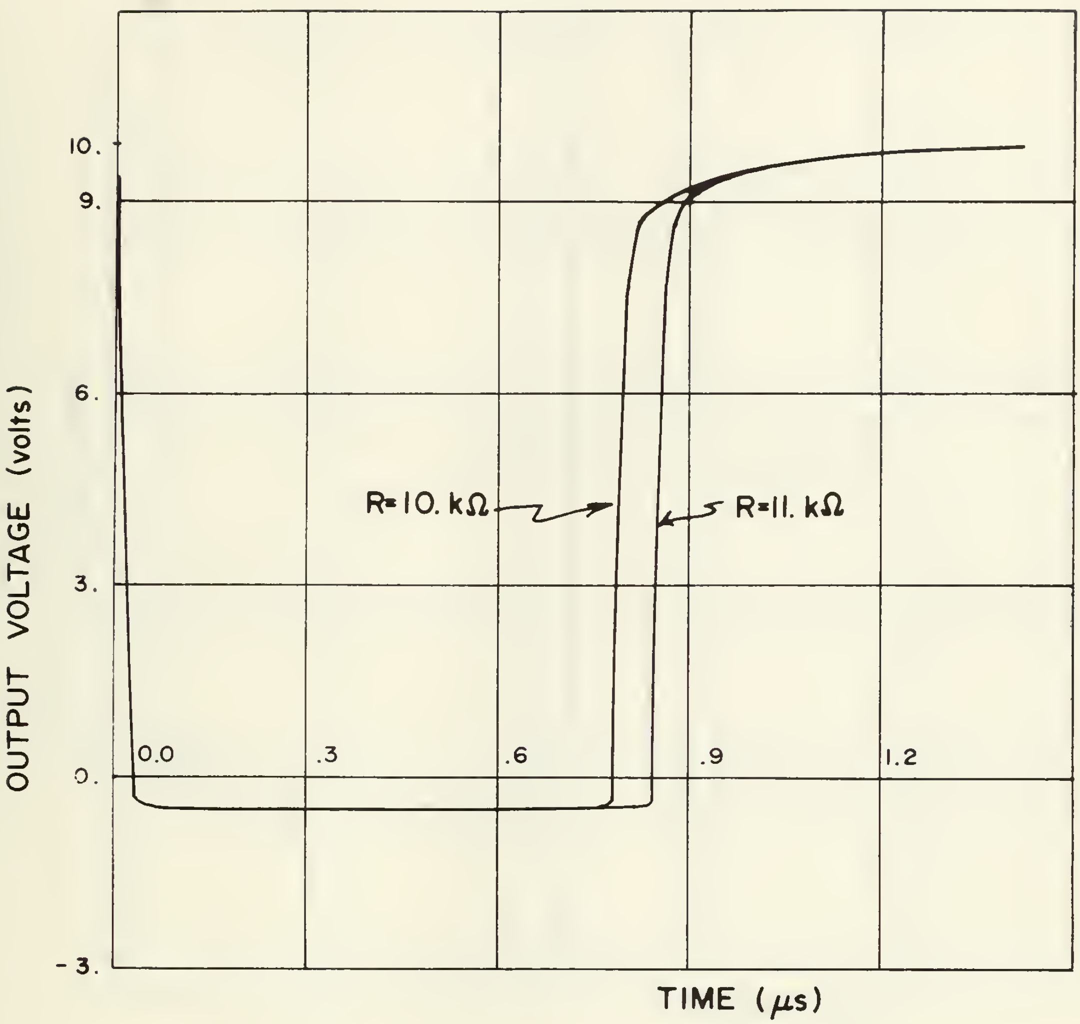


FIG. 4.6. RESPONSE OF DIODE CIRCUIT TO RADIATION PULSE.

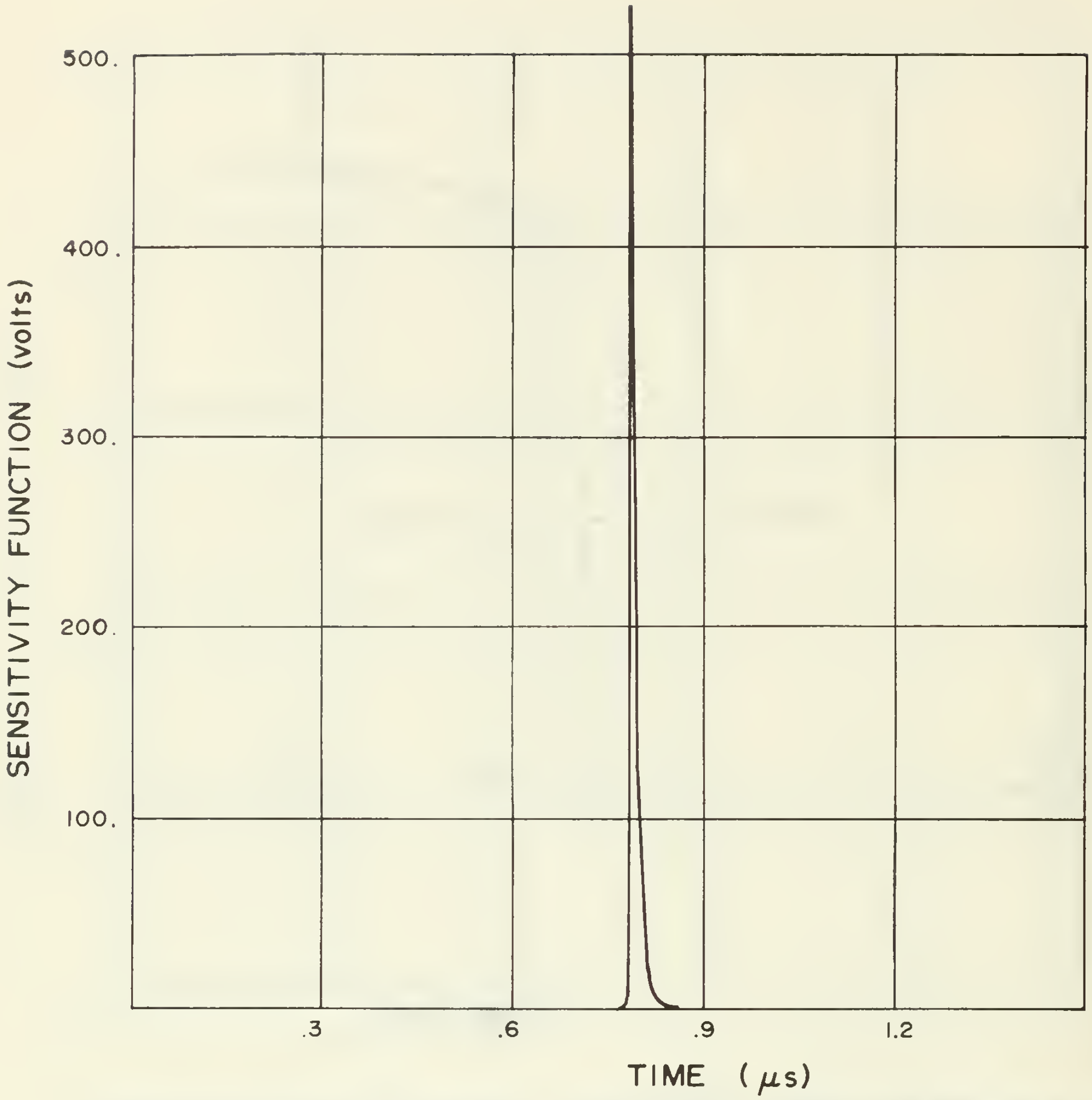


FIG. 4.7. SENSITIVITY FUNCTION WITH RESPECT TO A.

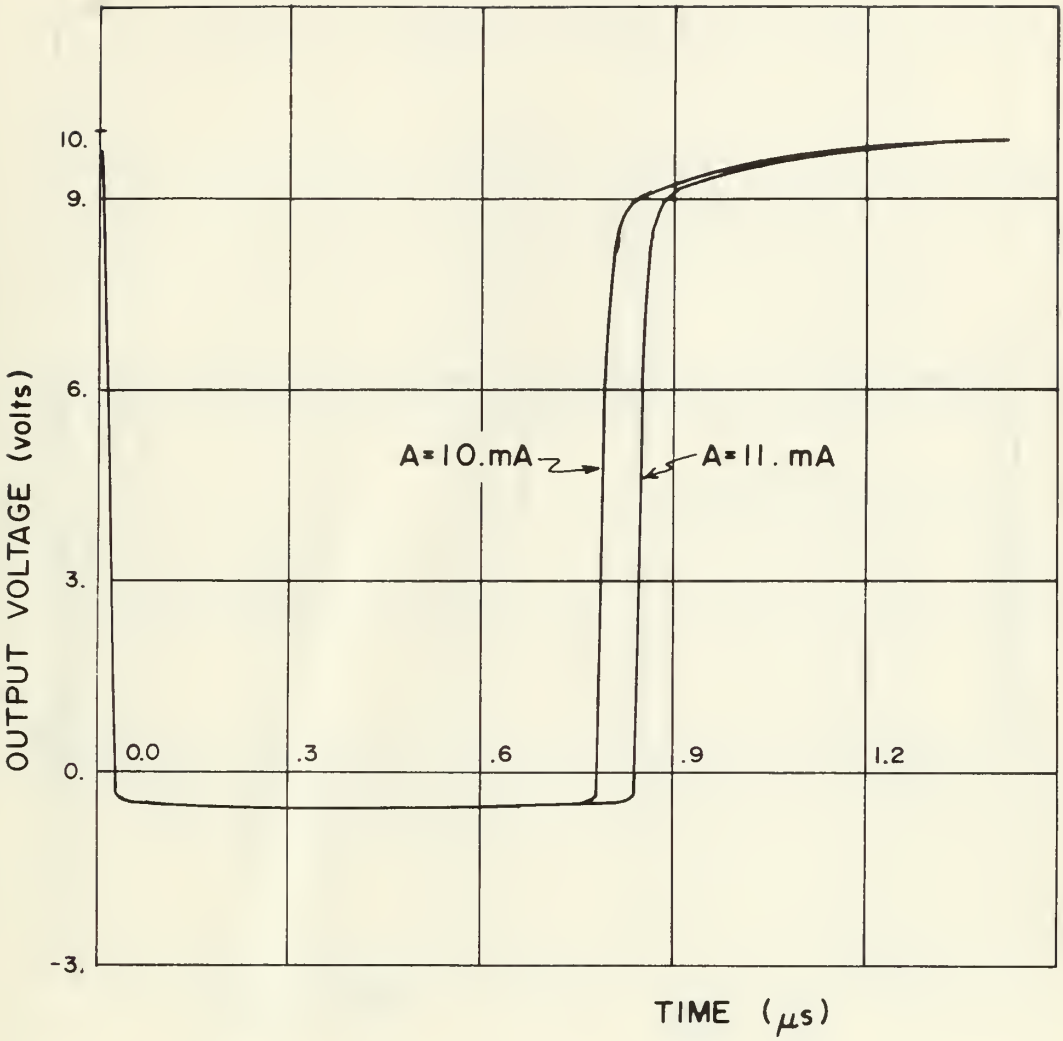


FIG. 4.8. RESPONSE OF DIODE CIRCUIT TO RADIATION PULSE.

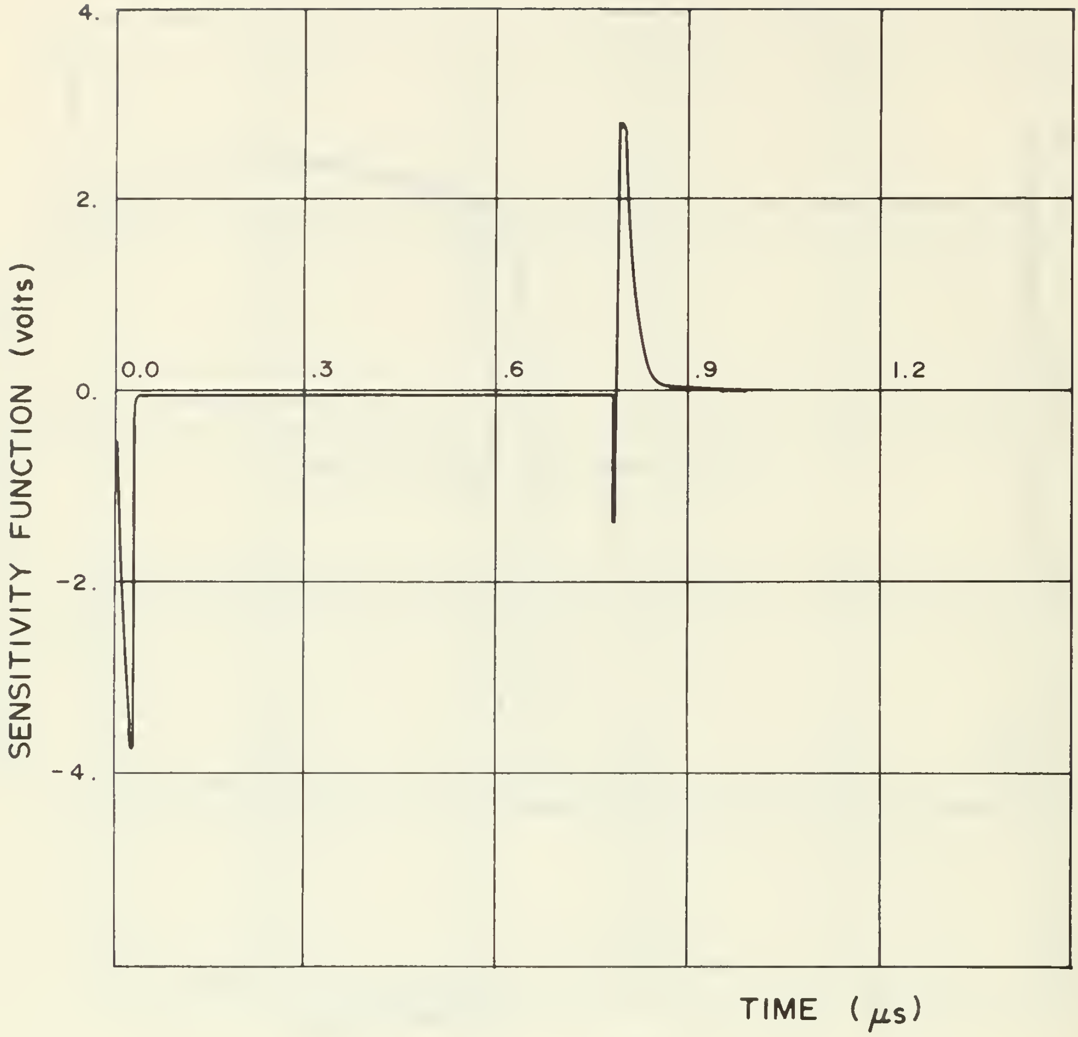


FIG. 4.9. SENSITIVITY FUNCTION WITH RESPECT TO C_D.

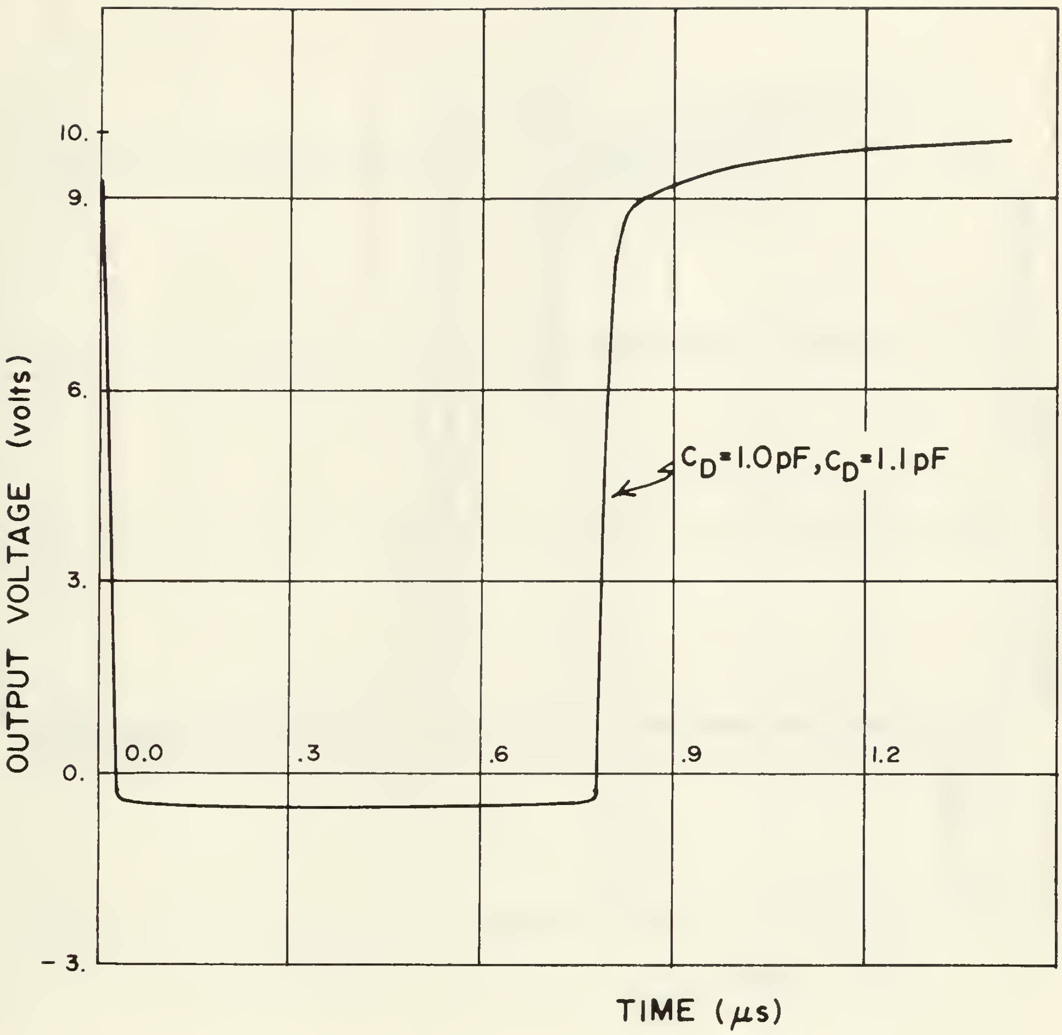


FIG. 4.10. RESPONSE OF DIODE CIRCUIT TO RADIATION PULSE.

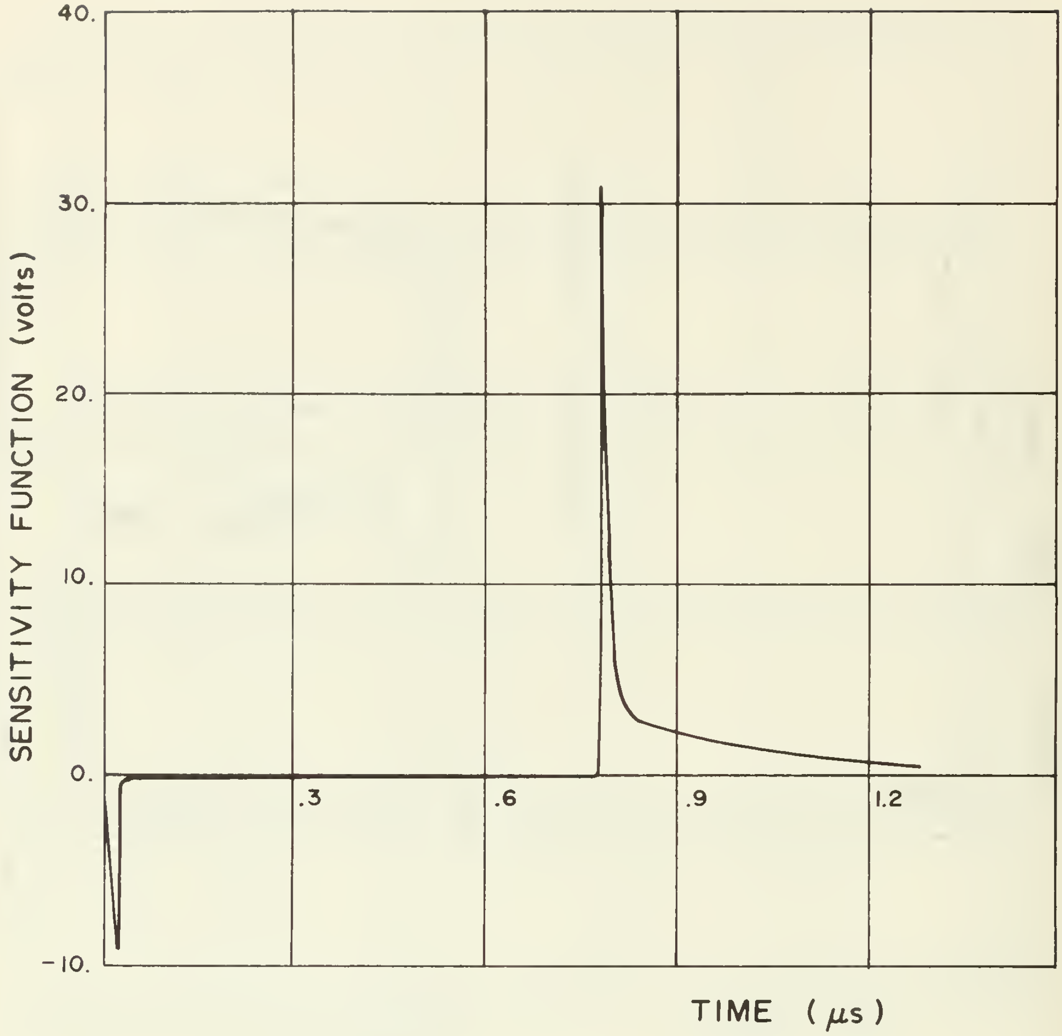


FIG. 4.11. SENSITIVITY FUNCTION WITH RESPECT TO T_R.

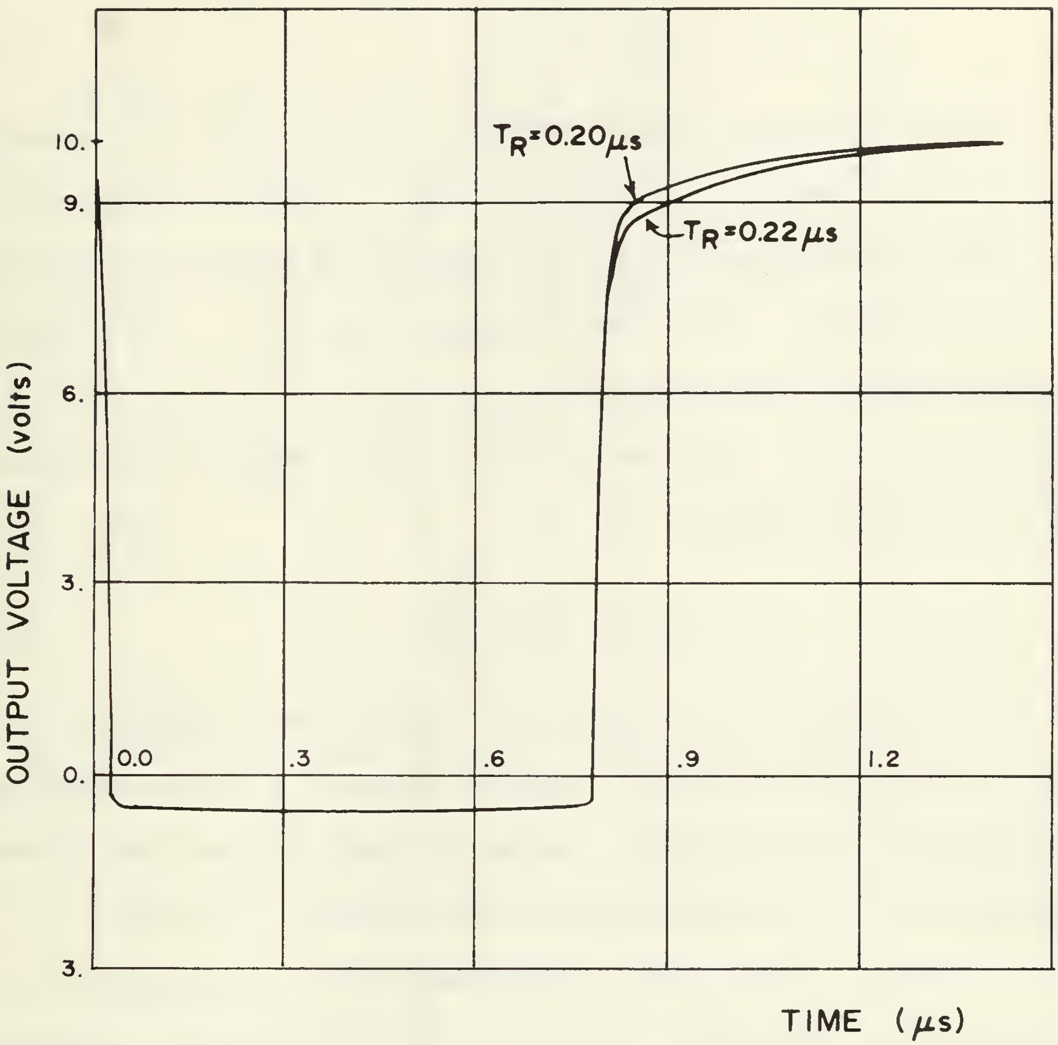


FIG. 4.12. RESPONSE OF DIODE CIRCUIT TO RADIATION PULSE.

be used to approximate changes in a response due to fractional changes in a parameter as follows

$$\Delta x_i = S_{a_j}^{x_i} \cdot \left(\frac{\Delta a_j}{a_j} \right) \qquad (4.14)$$

In general this approximation becomes more accurate as the percentage change in the parameter value approaches zero. Table 4.2 represents a comparison of results obtained for the underdamped response of the linear example in Chapter II. The output voltages, v_0 , v_1 , ... , v_5 , are obtained using resistance values of R_0, 1.1 R_0, ... , 1.5 R_0, where R_0 is the nominal value. Approximations for these voltages are calculated using (4.14); specifically,

$$v_a = v_0 + S_R^v \cdot \left(\frac{\Delta R_0}{R_0} \right) \qquad (4.15)$$

where v_a is the approximate response after R_0 has been changed by ΔR_0, and v_0 is the nominal response. Table 4.3 presents a similar comparison for the nonlinear diode example. Values are tabulated for time intervals when the sensitivity function does not vary rapidly. The results in tables 4.2 and 4.3 indicate that the approximation for the change in output voltage improves as the amplitude of the sensitivity function approaches a constant value. In this region correct values are predicted for variations as large as 50%. It is noted that (4.14) does not yield useful results for parameter variations as small as 1% when the sensitivity function displays sharp peaks. In this region the sensitivity function may still be used to determine changes in circuit responses by assuming that the associated parameter variations cause a time shift rather than a voltage change. Fig. 4.13a represents an idealized switching-circuit response where the effect of an incremental parameter variation is upon the switching time. Fig. 4.13b represents the corresponding sensitivity function.

Time ($\times 10^{-2}$ sec)	Sensitivity Function $\frac{\partial v}{\partial \ln R}$ (volts)	v_o	$\frac{\Delta R_o}{R_o} = .1$		$\frac{\Delta R_o}{R_o} = .2$		$\frac{\Delta R_o}{R_o} = .3$		$\frac{\Delta R_o}{R_o} = .4$		$\frac{\Delta R_o}{R_o} = .5$	
			v_1	v_a	v_2	v_a	v_3	v_a	v_4	v_a	v_5	v_a
.32	-22.09	30.89	28.82	28.68	27.01	26.47	25.40	24.26	23.97	22.05	22.69	19.84
.608	-20.10	38.24	36.31	36.23	34.55	34.22	32.93	32.21	31.44	30.20	30.08	28.19
.912	-13.83	32.47	31.14	31.09	29.90	29.70	28.74	28.32	27.66	26.94	26.65	25.55
1.2	-11.04	21.28	20.24	20.18	19.34	19.07	18.53	17.97	17.79	16.86	17.13	15.76
2.48	-11.03	-6.19	-7.17	-7.29	-8.06	-8.40	-8.58	-9.50	-9.08	-10.60	-9.47	-11.71
4.23	4.42	1.02	1.45	1.46	1.87	1.90	2.24	2.35	2.58	2.79	2.90	3.23

Table 4.2. Voltage comparison for linear example using (1) actual parameter changes, (2) approximation using (4.15).

Time (μs)	Sensitivity Function $\frac{\partial v}{\partial \ln T_R}$ (volts)	v_o	$\frac{\Delta T_R}{T_R} = .1$ v_1	v_a	$\frac{\Delta T_R}{T_R} = .2$ v_2	v_a	$\frac{\Delta T_R}{T_R} = .5$ v_5	v_a
.84	3.0	8.91	8.59	8.61	8.27	8.31	7.90	7.41
.94	2.0	9.37	9.14	9.17	8.91	8.97	8.19	8.37
1.13	1.0	9.76	9.65	9.66	9.52	9.56	9.06	9.26

Time (μs)	Sensitivity Function $\frac{\partial v}{\partial \ln E}$ (volts)	v_o	$\frac{\Delta E}{E} = .1$ v_1	v_a	$\frac{\Delta E}{E} = .2$ v_2	v_a	$\frac{\Delta E}{E} = .5$ v_5	v_a
.85	-10.0	9.02	10.03	10.02	11.03	11.02	14.03	14.02
.90	-10.0	9.24	10.24	10.24	11.24	11.24	14.24	14.24
1.20	-10.0	9.83	10.83	10.83	11.83	11.83	14.83	14.53

Table 4.3. Voltage comparison for nonlinear diode example using (1) actual parameter changes, (2) approximation using (4.15).

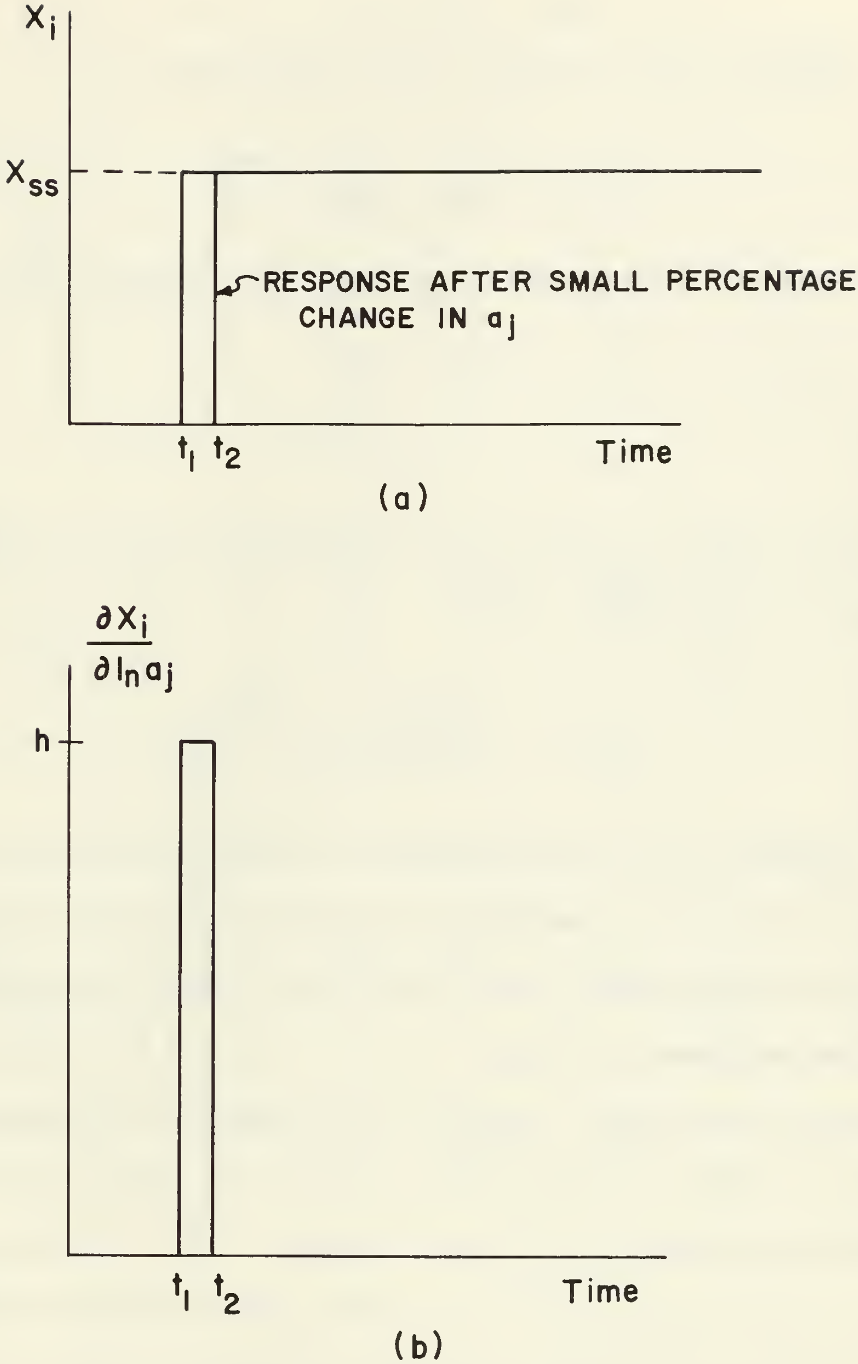


FIG. 4.13 (a) IDEALIZED CIRCUIT RESPONSE AND (b) ASSOCIATED SENSITIVITY FUNCTION.

The delay time, T_d, due to the parameter variation is given as

$$T_d = t_2 - t_1 \tag{4.16}$$

Applying (4.14) to Fig. 4.13 it follows that

$$\Delta x_i = x_{ss} = h \cdot \left(\frac{\Delta a_j}{a_j} \right) \tag{4.17}$$

The area, A, under the sensitivity function over the time interval $t_2 - t_1$ is

$$A = \int_{t_1}^{t_2} S_{a_j}^{x_i} \, dt = h \, T_d \tag{4.18}$$

Solving for h in (4.17) and substituting into (4.18) yields an equation for the delay time T_d.

$$T_d = \frac{(\Delta a_j/a_j)}{x_{ss}} \int_{t_1}^{t_2} S_{a_j}^{x_i} \, dt \tag{4.19}$$

Equation (4.19) may be applied to estimate the delay time corresponding to a pulse-type sensitivity function by calculating the area of the pulse and substituting into (4.19). This type of calculation is performed for the waveforms of Figures 4.1, 4.3, 4.5, and 4.7 for 10%, 20% and 50% parameter variations. The results are compared with the actual time delays in table 4.4. The area, $A = \int_{t_1}^{t_2} S_{a_j}^{x_i} \, dt$, is calculated assuming that the amplitude spikes are triangular. Times t_1 and t_2 are chosen at those points where $S_{a_j}^{x_i} = .15\,h$. The results indicate time delays can be calculated within 10% for parameter variation as large as 50%. It is noted that calculation of the area presents the greatest limitation in the accuracy of the delay time.

Parameter	$\frac{\Delta a_j}{a_j} = .1$ Calculated T_d (nsec)	Actual T_d (nsec)	$\frac{\Delta a_j}{a_j} = .2$ Calculated T_d (nsec)	Actual T_d (nsec)	$\frac{\Delta a_j}{a_j} = .5$ Calculated T_d (nsec)	Actual T_d (nsec)
T_D	11.0	9.6	21.0	19.2	43.0	48.0
E	-54.0	-50.0	-103.0	-100.0	-225.0	-250.0
R	58.0	49.5	113.0	99.0	258.0	247.5
A	58.0	49.5	112.0	99.0	257.0	247.5

Table 4.4. Comparison of actual and calculated delay times for given percentage changes in parameter values.

V. CONCLUSIONS

The technique of generating sensitivity functions for linear circuits using the concept of a sensitivity model is shown to be an extension of the compensation theorem. A set of iterative equations is developed to yield simultaneously the time responses of the circuit states and of their corresponding sensitivities. An example illustrates the procedure.

Diode and transistor modeling for discrete computation based upon the Ebers-Moll equations is considered. The models adopted contain both nonlinear capacitors and nonlinear resistors for the junctions. A state-variable formulation is developed which results in a set of nonlinear iterative equations for circuits containing linear elements, diodes, and transistors. The use of these equations is illustrated by considering the effects of a radiation pulse on a p-n junction, and the resulting recovery times of the junction are tabulated.

The foregoing nonlinear iterative equations are extended to generate simultaneously the sensitivity functions as well as the circuit solution. The preceding example, which considers the effects of a radiation pulse on a p-n junction, is then analyzed from the point of view of a sensitivity analysis. This analysis indicates that proper interpretation of the sensitivity function results in accurate estimated for changes in circuit responses due to parameter variations as large as 50%. When the amplitude of the sensitivity function is constant the change in the circuit response due to a parameter variation is reflected as a change in response amplitude. However, in regions where the sensitivity function displays impulse-like amplitude peaks, the change in the circuit response due to a parameter variation is reflected as a time delay or shift in the response characteristic.

APPENDIX A

PROOF OF $(\underline{\Delta x}^T \nabla)\,\underline{f} = (\nabla \underline{f}^T)^T\,\underline{\Delta x}$

The matrix equations $\underline{\Delta x}^T \nabla \underline{f}$ may be written as

$$(\underline{\Delta x}^T \nabla)\,\underline{f} = \left[\, \Delta x_1 \frac{\partial}{\partial x_1} + \Delta x_2 \frac{\partial}{\partial x_2} + \;.\;.\;.\; + \Delta x_n \frac{\partial}{\partial x_n} \,\right] \begin{bmatrix} f_1 \\ f_2 \\ \cdot \\ \cdot \\ \cdot \\ f_n \end{bmatrix}$$

$$= \begin{bmatrix} \Delta x_1 \frac{\partial f_1}{\partial x_1} + \Delta x_2 \frac{\partial f_1}{\partial x_2} + \;.\;.\;.\; + \Delta x_n \frac{\partial f_1}{\partial x_n} \\ \Delta x_1 \frac{\partial f_2}{\partial x_1} + \Delta x_2 \frac{\partial f_2}{\partial x_2} + \;.\;.\;.\; + \Delta x_n \frac{\partial f_2}{\partial x_n} \\ \cdot \\ \cdot \\ \cdot \\ \Delta x_1 \frac{\partial f_n}{\partial x_1} + \Delta x_2 \frac{\partial f_n}{\partial x_2} + \;.\;.\;.\; + \Delta x_n \frac{\partial f_n}{\partial x_n} \end{bmatrix} \qquad (A.1)$$

Consider this matrix equation

$$\nabla \underline{f}^T = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \cdot \\ \cdot \\ \cdot \\ \frac{\partial}{\partial x_n} \end{bmatrix} [f_1 \;\;.\;.\;.\;\; f_n] = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_2}{\partial x_1} & .\;.\;. & \frac{\partial f_n}{\partial x_1} \\ \frac{\partial f_1}{\partial x_2} & \frac{\partial f_2}{\partial x_2} & .\;.\;. & \frac{\partial f_n}{\partial x_2} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_1}{\partial x_n} & \frac{\partial f_2}{\partial x_n} & .\;.\;. & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \qquad (A.2)$$

Taking the transpose of equation (A.2) and multiplying by $\underline{\Delta x}$ yields

$$(\nabla \underline{f}^T)^T \, \underline{\Delta x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix}$$

$$= \begin{bmatrix} \Delta x_1 \frac{\partial f_1}{\partial x_1} + \Delta x_2 \frac{\partial f_1}{\partial x_2} + \cdots + \Delta x_n \frac{\partial f_1}{\partial x_n} \\ \Delta x_1 \frac{\partial f_2}{\partial x_1} + \Delta x_2 \frac{\partial f_2}{\partial x_2} + \cdots + \Delta x_n \frac{\partial f_2}{\partial x_n} \\ \vdots \\ \Delta x_1 \frac{\partial f_n}{\partial x_1} + \Delta x_2 \frac{\partial f_n}{\partial x_2} + \cdots + \Delta x_n \frac{\partial f_n}{\partial x_n} \end{bmatrix} \tag{A.3}$$

Equating equation (A.3) to equation (A.1) results in equation (A.4), which completes the proof.

$$(\underline{\Delta x}^T \nabla) \, \underline{f} \equiv (\nabla \underline{f}^T)^T \, \underline{\Delta x} \tag{A.4}$$

APPENDIX B

COMPUTER PROGRAMS

1. SOLUTION AND SENSITIVITY ANALYSIS PROGRAM FOR LINEAR CIRCUITS

```
      REAL*8 ITITLE
      DIMENSION  AI(2,2),AS(2,2),AMI(2,2),API(2,2),
     1AMIINV(2,2),PHI(2,2),AMIT2(2,2),BS(2,1),
     2GAM(2,1),BS1(2,1),BS2(2,1),BS3(2,1),
     3GAM21(2,1),GAM22(2,1),GAM23(2,1),GAM11(2,2),
     4GAM12(2,1),GAM13(2,2),TEMP1(2,2),TEMP2(2,2),
     5TEMP3(2,2),TEMP4(2,2),US(1,1),XS(2,1),XS1(2,1),
     6XS2(2,1),XS3(2,1),TEMPS1(2,1),TEMPS2(2,1),
     7TEMPS3(2,1),TEMPXS(2,1),S1(910),S2(910),S3(910),
     8XP(910),YP(910),AS1(2,2),AS2(2,2),AS3(2,2),
     9ITITLE(12)
      TIME=0.0
C     US MATRIX = 2*U
      US(1,1)=140.
C     EQUATIONS OF THE FORM
C     X(NT)    = PHI*X((N-1)T)    + GAM*(U(NT)+U((N-1)T))
C     XSJ(NT)  = PHI*XSJ((N-1)T)  + GAM*(USJ(NT)+USJ((N-1)T))
C      + GAM1J*(X(NT)+X((N-1)T)   + GAM2J*(U(NT)+U((N-1)T))
C     WHERE
C            (I-(T/2)*A)   =  AMI,   (I+(T/2)*A)   =  API
C                        -1
C            (I-(T/2)*A)   =  AMIINV
C                        -1
C            (I-(T/2)*A)   (I+(T/2)*A)   =   PHI
C                        -1
C            (I-(T/2)*A)   (B)*(T/2)     =   GAM
C                        -1
C            (I-(T/2)*A)   (ASJ*(T/2)    =   GAM1J
C                        -1
C            (I-(T/2)*A)   (BSJ)*(T/2)   =   GAM2J
C     NS  =  # OF STATES
C     NU  =  # OF FORCING FUNCTIONS
C     FORMAT AND DIMENSION STATEMENTS MUST BE CHANGED WHEN
C     THE NUMBER OF STATES OR FORCING FUNCTIONS ARE CHANGED
      NS=2
      NU=1
C     READ IN THE I AND A MATRICES
      READ(5,1)  ((AI(I,J),J=1,NS),I=1,NS)
      READ(5,1)  ((AS(I,J),J=1,NS),I=1,NS)
    1 FORMAT(8E10.2)
C     CALCULATE
C            AMI  =  (I-(T/2)A)        API   =  (I+(T/2)A)
      T=.16E-03
      Q=T/2.
      CALL CONST(Q,AS,NS,NS,TEMP1)
      CALL SUB(AI,TEMP1,NS,NS,AMI)
      CALL ADD(AI,TEMP1,NS,NS,API)
C     CALCULATE                      -1
C            AMIINV   =   (I-(T/2)A)
      CALL RECIP(.000001,AMI,AMIINV,KER,NS)
      IF(KER-2) 2,3,2
    3 WRITE(6,4)
    4 FORMAT(5HKER=2)
      GO TO 40
    2 CONTINUE
C     CALCULATE  PHI = AMIINV*API
      CALL PROD(AMIINV,API,NS,NS,NS,PHI)
C     CALCULATE  AMIT2 = AMIINV*(T/2)
      CALL CONST(Q,AMIINV,NS,NS,AMIT2)
C     READ IN B MATRIX
      READ(5,1)  ((BS(I,J),J=1,NU),I=1,NS)
C     CALCULATE  GAM = AMIT2*B
```

```
      CALL PROD(AMIT2,BS,NS,NS,NU,GAM)
C     READ IN BS MATRICES AND CALCULATE GAM2J = AMIT2*BSJ
      READ(5,1) ((BS1(I,J),J=1,NU),I=1,NS)
      READ(5,1) ((BS2(I,J),J=1,NU),I=1,NS)
      READ(5,1) ((BS3(I,J),J=1,NU),I=1,NS)
      CALL PROD(AMIT2,BS1,NS,NS,NU,GAM21)
      CALL PROD(AMIT2,BS2,NS,NS,NU,GAM22)
      CALL PROD(AMIT2,BS3,NS,NS,NU,GAM23)
C     READ IN ASJ MATRICES AND CALCULATE GAM1J = AMIT2*ASJ
      READ(5,1) ((AS1(I,J),J=1,NS),I=1,NS)
      READ(5,1) ((AS2(I,J),J=1,NS),I=1,NS)
      READ(5,1) ((AS3(I,J),J=1,NS),I=1,NS)
      CALL PROD(AMIT2,AS1,NS,NS,NS,GAM11)
      CALL PROD(AMIT2,AS2,NS,NS,NS,GAM12)
      CALL PROD(AMIT2,AS3,NS,NS,NS,GAM13)
      WRITE(6,12)
   12 FORMAT('1',21X,'A MATRIX')
      WRITE(6,14) ((AS(I,J),J=1,NS),I=1,NS)
   14 FORMAT(//,2(10X,E10.2),/)
      WRITE(6,15)
   15 FORMAT(//22X,'B MATRIX')
      WRITE(6,16) ((BS(I,J),J=1,NU),I=1,NS)
   16 FORMAT(//,1(21X,E10.2,/))
      WRITE(6,13)
   13 FORMAT(//22X,'PHI MATRIX')
      WRITE(6,14) ((PHI(I,J),J=1,NS),I=1,NS)
      WRITE(6,24)
   24 FORMAT(//22X,'GAM MATRIX')
      WRITE(6,16) ((GAM(I,J),J=1,NU),I=1,NS)
      WRITE(6,17)
   17 FORMAT(//,T27,'SENSITIVITY MATRICES',/,T19,'GAM11',T49
     1,'GAM21')
      WRITE(6,19) (((GAM11(I,J),J=1,NS),(GAM21(I,J),J=1,NU))
     1,I=1,NS)
   19 FORMAT(//,2(5X,E10.2,5X),5X,E10.2,/)
      WRITE(6,20)
   20 FORMAT(///,T19,'GAM12',T49,'GAM22')
      WRITE(6,19) (((GAM12(I,J),J=1,NS),(GAM22(I,J),J=1,NU))
     1,I=1,NS)
      WRITE(6,22)
   22 FORMAT(///,T19,'GAM13',T49,'GAM23')
      WRITE(6,19) (((GAM13(I,J),J=1,NS),(GAM23(I,J),J=1,NU))
     1,I=1,NS)
      WRITE(6,31)
   31 FORMAT('1',//,T13,'OUTPUT',T30,'SENSITIVITY',T50,'SENS
     1ITIVITY',T70,'SENSITIVITY',/,T33,'WRT R',T53,'WRT L',T
     273,'WRT C',T93,'TIME',///)
C      INITIALIZE
      READ(5,26) ((XS(I,1),I=1,NS),(XS1(I,1),I=1,NS),(XS2(I,
     11),I=1,NS),(XS3(I,1),I=1,NS))
      READ(5,26) XP(1),YP(1),S1(1),S2(1),S3(1)
   26 FORMAT(8E10.2)
      I=1
      Q=1.
   30 WRITE(6,32) XS(1,1),XS1(1,1),XS2(1,1),XS3(1,1),TIME
   32 FORMAT(5F20.5)
      CALL CONST(Q,XS,NS,NU,TEMPXS)
      CALL CONST(Q,XS1,NS,NU,TEMPS1)
      CALL CONST(Q,XS2,NS,NU,TEMPS2)
      CALL CONST(Q,XS3,NS,NU,TEMPS3)
C     CALCULATE STATE VECTOR
      CALL PROD(PHI,TEMPXS,NS,NS,NU,TEMP1)
      CALL PROD(GAM,US,NS,NU,NU,TEMP2)
      CALL ADD(TEMP1,TEMP2,NS,NU,XS)
      CALL ADD(XS,TEMPXS,NS,NU,TEMP4)
C     CALCULATE SENSITIVITY VECTORS
      CALL PROD(PHI,TEMPS1,NS,NS,NU,TEMP1)
      CALL PROD(GAM11,TEMP4,NS,NS,NU,TEMP2)
      CALL PROD(GAM21,US,NS,NU,NU,TEMP3)
      CALL ADD(TEMP1,TEMP2,NS,NU,TEMP1)
      CALL ADD(TEMP1,TEMP3,NS,NU,XS1)
      CALL PROD(PHI,TEMPS2,NS,NS,NU,TEMP1)
```

```
      CALL PPOD(GAM12,TEMP4,NS,NS,NU,TEMP2)
      CALL PROD(GAM22,US,NS,NU,NU,TEMP3)
      CALL ADD(TEMP1,TEMP2,NS,NU,TEMP1)
      CALL ADD(TEMP1,TEMP3,NS,NU,XS2)
      CALL PROD(PHI,TEMPS3,NS,NS,NU,TEMP1)
      CALL PROD(GAM13,TEMP4,NS,NS,NU,TEMP2)
      CALL PROD(GAM23,US,NS,NU,NU,TEMP3)
      CALL ADD(TEMP1,TEMP2,NS,NU,TEMP1)
      CALL ADD(TEMP1,TEMP3,NS,NU,XS3)
      I=I+1
      TIME=TIME+T
      XP(I)=TIME
      YP(I)=XS(1,1)
      S1(I)=XS1(1,1)
      S2(I)=XS2(1,1)
      S3(I)=XS3(1,1)
      IF(I.LT.905) GO TO 30
C     PLOT THE DATA POINTS
      READ(5,50) ITITLE
   50 FORMAT(6A8)
      READ(5,51) LABEL1
   51 FORMAT(A4)
      CALL DRAW(900,XP,YP,0,0,LABEL1,ITITLE,3.E-02,10.,
     11,0,2,2,5,6,1,L)
      WRITE(6,52) L
   52 FORMAT(//,1X,'L=',I10)
      READ(5,50) ITITLE
      CALL DRAW(900,XP,S3,1,0,LABEL1,ITITLE,3.E-02,10.,
     12,0,2,2,5,6,1,L)
      WRITE(6,52) L
      CALL DRAW(900,XP,S2,2,0,LABEL1,ITITLE,3.E-02,10.,
     12,0,2,2,5,6,1,L)
      WRITE(6,52) L
      CALL DRAW(900,XP,S1,3,0,LABEL1,ITITLE,3.E-02,10.,
     12,0,2,2,5,6,1,L)
      WRITE(6,52) L
   40 RETURN
      END
```

SUBROUTINES

```
      SUBROUTINE ADD (A,B,N,M,C)
C     THIS SUBROUTINE ADDS TWO NXM MATRICES
      DIMENSION A(N,M),B(N,M),C(N,M)
      DO 152 I=1,N
      DO 152 J=1,M
  152 C(I,J)=A(I,J)+B(I,J)
      RETURN
      END


      SUBROUTINE SUB (A,B,N,M,C)
C     THIS SUBROUTINE SUBTRACTS TWO NXM MATRICES
      DIMENSION A(N,M),B(N,M),C(N,M)
      DO 152 I=1,N
      DO 152 J=1,M
  152 C(I,J)=A(I,J)-B(I,J)
      RETURN
      END


      SUBROUTINE CONST(Q,A,N,M,C)
C     THIS SUBROUTINE MULTIPLIES A NXM MATRIX BY A CONSTANT
      DIMENSION A(N,M),C(N,M)
   15 DO 150 I=1,N
      DO 150 J=1,M
  150 C(I,J)=Q*A(I,J)
      RETURN
      END
```

```
      SUBROUTINE PROD (A,B,N,M,L,C)
C     THIS SUBROUTINE POST MULTIPLIES A NXM MATRIX
C     BY A MXL MATRIX
      DIMENSION A(N,M),B(M,L),C(N,L)
      DO 1 I=1,N
      DO 1 J=1,L
    1 C(I,J)=0.
      DO 151 I=1,N
      DO 151 J=1,L
      DO 151 K=1,M
  151 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN
      END


      SUBROUTINE RECIP(EP,A,X,KER,N)
C     THIS SUBROUTINE TAKES THE INVERSE OF A NXN MATRIX
      DIMENSION A(N,N),X(N,N)
      DO 1 I=1,N
      DO 1 J=1,N
    1 X(I,J)=0.
      DO 2 K=1,N
    2 X(K,K)=1.
   10 DO 34 L=1,N
      KP=0
      Z=0.
      DO 12 K=L,N
      IF(Z.GE.ABS(A(K,L))) GO TO 12
   11 Z=ABS(A(K,L))
      KP=K
   12 CONTINUE
      IF(L.GE.KP) GO TO 20
   13 DO 14 J=L,N
      Z=A(L,J)
      A(L,J)=A(KP,J)
   14 A(KP,J)=Z
      DO 15 J=1,N
      Z=X(L,J)
      X(L,J)=X(KP,J)
   15 X(KP,J)=Z
   20 IF(ABS(A(L,L)).LE.EP) GO TO 50
   30 IF(L.GE.N) GO TO 34
   31 LP1=L+1
      DO 36 K=LP1,N
      IF(A(K,L).EQ.0.) GO TO 36
   32 RATIO=A(K,L)/A(L,L)
      DO 33 J=LP1,N
   33 A(K,J)=A(K,J)-RATIO*A(L,J)
      DO 35 J=1,N
   35 X(K,J)=X(K,J)-RATIO*X(L,J)
   36 CONTINUE
   34 CONTINUE
   40 DO 43 I=1,N
      II=N+1-I
      DO 43 J=1,N
      S=0.
      IF(II.GE.N) GO TO 43
   41 IIP1=II+1
      DO 42 K=IIP1,N
   42 S=S+A(II,K)*X(K,J)
   43 X(II,J)=(X(II,J)-S)/A(II,II)
      KER=1
      RETURN
   50 KER=2
      RETURN
      END
```

2. DIODE SOLUTION USING PIECEWISE LINEAR MODEL

```
      DIMENSION V(600),CURR(600),YP(905),XP(905)
      REAL LABEL/4H    /
      REAL*8 ITITLE(12)/'          DIODE SOLUTION
     1  PIECEWISE LINEAR MODEL   T=1.E-09      '/
      ICNT=1
      IEND=5
      READ(5,3) SI,E,CD,RSH,R,TR,TD,RADC
    3 FORMAT(8E10.2)
  100 TIME=0.0
      READ(5,300) T,RD,NSTOP,V(1),XP(1),YP(1),CURR(1),TEMT
  300 FORMAT(2E10.2,I10,5E10.2)
      ALP=1./.026
      GOLD=RADC
      KKK=590
      KKKIN=KKK-1
      NM1=1
    6 N=1
    4 IF(TIME.LT.1.E-07) GO TO 1
      GNEW=0.0
      GO TO 2
    1 GNEW=RADC
    2 N=N+1
      CURR(N)=((2.*TR-T)/(2.*TR+T))*CURR(NM1)+(T/(2.*TR+T))*
     1(GOLD+GNEW)
      IF(V(NM1).LT.0.0) GO TO 9
      G=1./R+1./RSH+1./RD
      CAP=TD/RD
      GO TO 5
    9 G=1./R+1./RSH
      CAP=CD
    5 V(N)=((2.*CAP-G*T)*V(NM1)+T*(CURR(N)+CURR(NM1)-2.*E/R)
     1)/(2.*CAP+G*T)
      GOLD=GNEW
      TIME=TIME+T
      IF(N.GE.NSTOP) GO TO 20
      TEMT=TEMT+T
      IF(TEMT.GE..15E-08) GO TO 20
      NM1=N
      GO TO 4
   20 KKK=KKK+1
      II=KKK-KKKIN
      XP(II)=TIME
      YP(II)=-V(N)
      TEMT=0.
      NM1=N
      IF(KKK.LT.1493) GO TO 6
C     PLOT THE DATA POINTS
      IF(IEND.EQ.1) GO TO 104
      IF(ICNT.GT.1) GO TO 102
      CALL DRAW(900,XP,YP,1,0,LABEL1,ITITLE,3.E-07,3.,1,0,2,
     10,5,5,1,L)
      WRITE(6,13) L
   13 FORMAT(//,1X,'L=',I10)
      GO TO 103
  102 IF(ICNT.EQ.IEND) GO TO 101
      CALL DRAW(900,XP,YP,2,0,LABEL1,ITITLE,3.E-07,3.,1,0,2,
     10,5,5,1,L)
      WRITE(6,13) L
  103 ICNT=ICNT+1
      GO TO 100
  101 CALL DRAW(900,XP,YP,3,0,LABEL1,ITITLE,3.E-07,3.,1,0,2,
     10,5,5,1,L)
      WRITE(6,13) L
      GO TO 106
  104 CALL DRAW(900,XP,YP,0,0,LABEL1,ITITLE,3.E-07,3.,1,0,2,
     10,5,5,1,L)
      WRITE(6,13) L
  106 RETURN
      END
```

3. DIODE SOLUTION HOLDING DIODE CAPACITANCE AND RESISTANCE CONSTANT BETWEEN CALCULATION POINTS

```
      REAL*8 ITITLE
      DIMENSION V(600),CURR(600),XP(905),YP(905),
     1ITITLE(12)
      ICNT=1
      IEND=3
      READ(5,11) ITITLE
   11 FORMAT(6A8)
      READ(5,12) LABEL1
   12 FORMAT(A4)
      READ(5,3) SI,E,CD,RSH,R,TR,TD,RADC
    3 FORMAT(8F10.2)
  100 TIME=0.0
      CALL CANCEL(2)
      READ(5,300) T,GAM,NSTOP
  300 FORMAT(2E10.2,I10)
      TEMT=0.0
      ALP=1./.026
C     INITIALIZE RD AND CAP
      RD=1.E+50
      CAP=CD
      V(1)=-10.
      CURR(1)=0.0
      GOLD=RADC
      KKK=590
      KKKIN=KKK-1
      NM1=1
      N=1
    4 IF(TIME.LT.1.E-07) GO TO 1
      GNEW=0.0
      GO TO 2
    1 GNEW=RADC
    2 N=N+1
      T=2.*T
      IF(T.GE.1.E-09)T=1.E-09
    9 CURR(N)=((2.*TR-T)/(2.*TR+T))*CURR(NM1)+(T/(2.*TR+T))*
     1(GOLD+GNEW)
      G=1./R+1./RSH+1./RD
      V(N)=((2.*CAP-G*T)*V(NM1)+T*(CURR(N)+CURR(NM1)-2.*E/R)
     1)/(2.*CAP+G*T)
      RD=V(N)/(SI*(EXP(ALP*V(N))-1.))
      GTTEST=1./R+1./RSH+1./RD
      CTTEST=CD+TD*SI*ALP*EXP(ALP*V(N))
      CON1=ABS(G-GTTEST)
      CON2=ABS(CAP-CTTEST)
      CON3=GAM*GTTEST
      CON4=GAM*CTTEST
      IF(CON1.LT.CON3.AND.CON2.LT.CON4) GO TO 7
      IF(GAM.EQ.0.0) GO TO 7
      T=T/2.
      GO TO 9
    7 GOLD=GNEW
      CAP=CTTEST
      TIME=TIME+T
      IF(N.GE.NSTOP) GO TO 20
      TEMT=TEMT+T
      IF(TEMT.GE..16E-08) GO TO 20
      NM1=N
      GO TO 4
   20 KKK=KKK+1
      II=KKK-KKKIN
      XP(II)=TIME
      YP(II)=-V(N)
      NM1=N
      N=1
      TEMT=0.0
      IF(KKK.LT.1493) GO TO 4
```

```
      XP(1)=0.0
      YP(1)=10.
C     PLOT THE DATA POINTS
      IF(IEND.EQ.1) GO TO 104
      IF(ICNT.GT.1) GO TO 102
      CALL DRAW(900,XP,YP,1,0,LABEL1,ITITLE,3.E-07,3.,1,0,2,
     10,5,5,1,L)
      WRITE(6,13) L
   13 FORMAT(//,1X,'L=',I10)
      GO TO 103
  102 IF(ICNT.EQ.IEND) GO TO 101
      CALL DRAW(900,XP,YP,2,0,LABEL1,ITITLE,3.E-07,3.,1,0,2,
     10,5,5,1,L)
      WRITE(6,13) L
  103 ICNT=ICNT+1
      GO TO 100
  101 CALL DRAW(900,XP,YP,3,0,LABEL1,ITITLE,3.E-07,3.,1,0,2,
     10,5,5,1,L)
      WRITE(6,13) L
      GO TO 106
  104 CALL DRAW(900,XP,YP,0,0,LABEL1,ITITLE,3.E-07,3.,1,0,2,
     10,5,5,1,L)
      WRITE(6,13) L
  106 RETURN
      END
```

4. SOLUTION AND SENSITIVITY ANALYSIS PROGRAM FOR NONLINEAR DIODE EXAMPLE

```
      REAL*8 ITITLE
      DIMENSION ITITLE(12),CURR(600),XP(905),V(600),YP(905),
     1XS1(600),S1(905),XS2(600),S2(905),XS3(600),S3(905),
     2XS4(600),S4(905),SCUR(600),XS5(600),S5(905),XS6(600),
     3S6(905),TCUR(600)
      CALL CANCEL(2)
      NSTOP=300.
      READ(5,12) LABEL1
   12 FORMAT(A4)
      TIME=0.0
      READ(5,3)T,SI,E,R,TR,TD,RADC,CD
      READ(5,3) V(1),XP(1),YP(1),CURR(1),SCUR(1),TCUR(1)
      READ(5,3) VMIN,S1MIN,S2MIN,S3MIN,S4MIN,S5MIN,S6MIN
      READ(5,3) XS1(1),XS2(1),XS3(1),XS4(1),XS5(1),XS6(1)
      READ(5,3) S1(1),S2(1),S3(1),S4(1),S5(1),S6(1)
    3 FORMAT(8E10.2)
      TEMT=0.0
      ALP=1./.026
      RSH=1.E+08
      G=1./R+1./RSH
      UO=-E/R
      US1O=0.
      US2O=-E/R
      US3O=E/R
      US4O=0.
      US5O=0.
      US6O=0.
      US1N=0.
      US2N=-E/R
      US3N=E/R
      US5N=0.
      US1T=0.
      US2T=US2O+US2N
      US3T=US3O+US3N
      US5T=0.
      GOLD=RADC
      KKK=590
      KKKIN=KKK-1
      NM1=1
      N=1
    4 IF(TIME.LT.1.E-07) GO TO 1
```

```
      GNEW=0.0
      GO TO 2
    1 GNEW=RADC
    2 N=N+1
      T=2.*T
      IF(T.GE.1.E-10) T=1.E-10
      CON1=SI*ALP*EXP(ALP*V(NM1))
      CON2=TD*CON1
      CON3=ALP*CON2
      CON4=SI*(EXP(ALP*V(NM1))-1.)
      DEN=CD+CON2
      DEN2=DEN**2
      DEN3=DEN**3
      F=-(G*V(NM1)+CON4)/DEN
      B=1./DEN
      PF=-(G+CON1)/DEN+(CON3*(G*V(NM1)+CON4))/DEN2
      PB=-CON3/DEN2
      FS1=-((G+CON1)*XS1(NM1))/DEN+((CON4+G*V(NM1))*(CON2
     1+CON3*XS1(NM1)))/DEN2
      FS2=-((G+CON1)*XS2(NM1))/DEN+(CON3*XS2(NM1)*(G*V(NM1)+
     1CON4))/DEN2
      FS3=-((G+CON1)*XS3(NM1)-V(NM1)/R)/DEN+(CON3*XS3(NM1)*
     1(G*V(NM1)+CON4))/DEN2
      FS4=-((G+CON1)*XS4(NM1))/DEN+(CON3*XS4(NM1)*(G*V(NM1)+
     1CON4))/DEN2
      FS5=-((G+CON1)*XS5(NM1))/DEN+((G*V(NM1)+CON4)*(CD+CON3
     1*XS5(NM1)))/DEN2
      FS6=-((G+CON1)*XS6(NM1))/DEN+(CON3*XS6(NM1)*(G*V(NM1)+
     1CON4))/DEN2
      BS1=-(CON2+CON3*XS1(NM1))/DEN2
      BS2=-(CON3*XS2(NM1))/DEN2
      BS3=-(CON3*XS3(NM1))/DEN2
      BS4=-(CON3*XS4(NM1))/DEN2
      BS5=-(CD+CON3*XS5(NM1))/DEN2
      BS6=-(CON3*XS6(NM1))/DEN2
      PFS1=-(G+CON1)/DEN+(CON3*(CON4+G*V(NM1)))/DEN2
      PFS2=PFS1
      PFS3=PFS1
      PFS4=PFS1
      PFS5=PFS1
      PFS6=PFS1
      PXFS1=-(ALP*CON1*XS1(NM1))/DEN+((G+CON1)*(CON2+2.*CON3
     1*XS1(NM1))+(G*V(NM1)+CON4)*(CON3+ALP*CON3*XS1(NM1)))/
     2DEN2-(2.*CON3*(G*V(NM1)+CON4)*(CON2+CON3*XS1(NM1)))/
     3DEN3
      PXFS2=-(ALP*CON1*XS2(NM1))/DEN+(ALP*CON3*XS2(NM1)*(G*
     1V(NM1)+CON4)+2.*CON3*XS2(NM1)*(G+CON1))/DEN2-(2.*CON3*
     2CON3*XS2(NM1)*(G*V(NM1)+CON4))/DEN3
      PXFS3=-(ALP*CON1*XS3(NM1)-1./R)/DEN+(ALP*CON3*XS3(NM1)
     1*(G*V(NM1)+CON4)+2.*CON3*XS3(NM1)*(G+CON1)-CON3*
     2V(NM1)/R)/DEN2-(2.*CON3*CON3*(G*V(NM1)+CON4)*XS3(NM1))
     3/DEN3
      PXFS4=-(ALP*CON1*XS4(NM1))/DEN+(ALP*CON3*XS4(NM1)*(G*
     1V(NM1)+CON4)+2.*CON3*XS4(NM1)*(G+CON1))/DEN2-(2.*CON3*
     2CON3*XS4(NM1)*(G*V(NM1)+CON4))/DEN3
      PXFS5=-(ALP*CON1*XS5(NM1))/DEN+((G+CON1)*(CD+2.*CON3*
     1XS5(NM1))+(G*V(NM1)+CON4)*(ALP*CON3*XS5(NM1)))/DEN2-(
     22.*CON3*(G*V(NM1)+CON4)*(CD+CON3*XS5(NM1)))/DEN3
      PXFS6=-(ALP*CON1*XS6(NM1))/DEN+(ALP*CON3*XS6(NM1)*(G*
     1V(NM1)+CON4)+2.*CON3*XS6(NM1)*(G+CON1))/DEN2-(2.*CON3*
     2CON3*XS6(NM1)*(G*V(NM1)+CON4))/DEN3
      PBS1=-CON3/DEN2
      PBS2=PBS1
      PBS3=PBS1
      PBS4=PBS1
      PBS5=PBS1
      PBS6=PBS1
      PXBS1=-(CON3*(1.+ALP*XS1(NM1)))/DEN2+(2.*CON3*(CON2+
     1CON3*XS1(NM1)))/DEN3
      PXBS2=-(ALP*CON3*XS2(NM1))/DEN2+(2.*CON3*CON3*XS2(NM1)
     1)/DEN3
      PXBS3=-(ALP*CON3*XS3(NM1))/DEN2+(2.*CON3*CON3*XS3(NM1)
```

```
 1)/DEN3
  PXBS4=-(ALP*CON3*XS4(NM1))/DEN2+(2.*CON3*CON3*XS4(NM1)
 1)/DEN3
  PXBS5=-(ALP*CON3*XS5(NM1))/DEN2+(2.*CON3*(CD+CON3*
 1XS5(NM1)))/DEN3
  PXBS6=-(ALP*CON3*XS6(NM1))/DEN2+(2.*CON3*CON3*XS6(NM1)
 1)/DEN3
9 CURR(N)=((2.*TR-T)/(2.*TR+T))*CURR(NM1)+(T/(2.*TR+T))*
 1(GOLD+GNEW)
  SCUR(N)=((2.*TR-T)/(2.*TR+T))*SCUR(NM1)+(T/(2.*TR+T))*
 1(GOLD+GNEW)
  TCUR(N)=((2.*TR-T)*TCUR(NM1)+T*(CURR(N)+CURR(NM1)
 1-GOLD-GNEW))/(2.*TR+T)
  UN=CURR(N)-E/R
  US4N=SCUR(N)
  US6N=TCUR(N)
  UT=UN+UO
  US4T=US4N+US40
  US6T=US60+US6N
  V(N)=V(NM1)+(T*(F+.5*B*UT))/(1.-T*(PF+PB*UN)/2.)
  DELV=V(N)-V(NM1)
  VNORM= ABS(DELV)
  IF(VNORM.GT.VMIN) GO TO 5
  XS1(N)=XS1(NM1)+(T*(FS1+.5*(DELV*(PXFS1+PB*US1N+PXBS1*
 1UN)+B*US1T+BS1*UT)))/(1.-(T*(PFS1+PBS1*UN))/2.)
  DELS1=XS1(N)-XS1(NM1)
  S1NORM= SQRT(DELV**2+DELS1**2)
  IF(S1NORM.GT.S1MIN) GO TO 5
  XS2(N)=XS2(NM1)+(T*(FS2+.5*(DELV*(PXFS2+PB*US2N+PXBS2*
 1UN)+B*US2T+BS2*UT)))/(1.-(T*(PFS2+PBS2*UN))/2.)
  DELS2=XS2(N)-XS2(NM1)
  S2NORM= SQRT(DELV**2+DELS2**2)
  IF(S2NORM.GT.S2MIN) GO TO 5
  XS3(N)=XS3(NM1)+(T*(FS3+.5*(DELV*(PXFS3+PB*US3N+PXBS3*
 1UN)+B*US3T+BS3*UT)))/(1.-(T*(PFS3+PBS3*UN))/2.)
  DELS3=XS3(N)-XS3(NM1)
  S3NORM= SQRT(DELV**2+DELS3**2)
  IF(S3NORM.GT.S3MIN) GO TO 5
  XS4(N)=XS4(NM1)+(T*(FS4+.5*(DELV*(PXFS4+PB*US4N+PXBS4*
 1UN)+B*US4T+BS4*UT)))/(1.-(T*(PFS4+PBS4*UN))/2.)
  DELS4=XS4(N)-XS4(NM1)
  S4NORM= SQRT(DELV**2+DELS4**2)
  IF(S4NORM.GT.S4MIN) GO TO 5
  XS5(N)=XS5(NM1)+(T*(FS5+.5*(DELV*(PXFS5+PB*US5N+PXBS5*
 1UN)+B*US5T+BS5*UT)))/(1.-(T*(PFS5+PBS5*UN))/2.)
  DELS5=XS5(N)-XS5(NM1)
  S5NORM= SQRT(DELV**2+DELS5**2)
  IF(S5NORM.GT.S5MIN) GO TO 5
  XS6(N)=XS6(NM1)+(T*(FS6+.5*(DELV*(PXFS6+PB*US6N+PXBS6*
 1UN)+B*US6T+BS6*UT)))/(1.-(T*(PFS6+PBS6*UN))/2.)
  DELS6=XS6(N)-XS6(NM1)
  S6NORM= SQRT(DELV**2+DELS6**2)
  IF(S6NORM.GT.S6MIN) GO TO 5
  GO TO 7
5 T=T/2.
  GO TO 9
7 GOLD=GNEW
  TIME=TIME+T
  UO=UN
  US40=US4N
  US60=US6N
  IF(N.GE.NSTOP) GO TO 20
  TEMT=TEMT+T
  IF(TEMT.GE..15E-08) GO TO 20
  NM1=N
  GO TO 4
20 KKK=KKK+1
  II=KKK-KKKIN
  XP(II)=TIME
  YP(II)=-V(N)
  S1(II)=XS1(N)
  S2(II)=XS2(N)
```

```
      S3(II)=XS3(N)
      S4(II)=XS4(N)
      S5(II)=XS5(N)
      S6(II)=XS6(N)
      WRITE(6,14) TIME,CURR(N),V(N),XS1(N),XS2(N),XS3(N),
     1XS4(N),XS5(N),XS6(N),T,N
   14 FORMAT(10E12.4,I5)
      NM1=N
      N=1
      TEMT=0.0
      IF(KKK.LT.1493) GO TO 4
C     PLOT THE DATA POINTS
      READ(5,11) ITITLE
   11 FORMAT(6A8)
      CALL DRAW(900,XP,YP,0,0,LABEL1,ITITLE,3.E-07,3.,
     11,0,2,2,5,5,1,L)
      WRITE(6,13) L
   13 FORMAT(//,1X,'L=',I10)
C     PLOT THE SENSITIVITY FUNCTIONS
      READ(5,11) ITITLE
      CALL DRAW(900,XP,S1,0,0,LABEL1,ITITLE,3.E-07,20.,
     10,0,2,2,5,5,1,L)
      WRITE(6,13) L
      READ(5,11) ITITLE
      CALL DRAW(900,XP,S2,0,0,LABEL1,ITITLE,3.E-07,100.,
     15,0,2,2,5,5,1,L)
      WRITE(6,13) L
      READ(5,11) ITITLE
      CALL DRAW(900,XP,S3,0,0,LABEL1,ITITLE,3.E-07,100.,
     10,0,2,2,5,5,1,L)
      WRITE(6,13) L
      READ(5,11) ITITLE
      CALL DRAW(900,XP,S4,0,0,LABEL1,ITITLE,3.E-07,100.,
     10,0,2,2,5,5,1,L)
      WRITE(6,13) L
      READ(5,11) ITITLE
      CALL DRAW(900,XP,S5,0,0,LABEL1,ITITLE,3.E-07,2.,
     13,0,2,2,5,5,1,L)
      WRITE(6,13) L
      READ(5,11) ITITLE
      CALL DRAW(900,XP,S6,0,0,LABEL1,ITITLE,3.E-07,10.,
     11,0,2,2,5,5,1,L)
      WRITE(6,13) L
  106 RETURN
      END
```

5. TRANSISTOR BIASING PROGRAM

```
      CALL CANCEL(2)
C     READ IN THE TRANSISTOR PARAMETERS
      READ(5,2) RSHE,CDE,TDE,SIE,ALPE,ALPI
      READ(5,2) RSHC,CDC,TDC,SIC,ALPC,ALPN
C     READ IN THE DESIRED BASE-EMITTER, BASE-COLLECTOR AND
C     BIASING VOLTAGES
      READ(5,2) E,X1,X2
    2 FORMAT(6E10.2)
      GC=(X1/RSHE-ALPI*SIC*(EXP(ALPC*X2)-1.)+SIE*(EXP(ALPE*
     1X1)-1.))/(E+X2-X1)
      RC=1./GC
      GB=(GC*(X1-E-X2)-X2/RSHC+ALPN*SIE*(EXP(ALPE*X1)-1.)
     1-SIC*(EXP(ALPC*X2)-1.))/X2
      RB=1./GB
C     WRITE THE VALUES OF THE BIASING RESISTORS
      WRITE(6,5) RB,RC
    5 FORMAT('1',//2E15.5)
      RETURN
      END
```

6. SOLUTION PROGRAM FOR NONLINEAR TRANSISTOR EXAMPLE

```
      REAL*8 ITITLE
      DIMENSION F(2,1),DER(2,2),AI(2,2),TEMP(2,2),
     1V(905),XP(905),XS(2,1),TEMP1(2,2),DERINV(2,2),
     2B(2,2),U(2,1),BDER(2,2),UT(2,1),TEMP2(2,2),UTEMP(2,1)
     3,ITITLE(12)
      CALL CANCEL(2)
      NS=2
      NU=1
      READ(5,20) ((AI(I,J),J=1,NS),I=1,NS)
   20 FORMAT(8E10.2)
      READ(5,2) RSHE,CDE,RB ,SIE,ALPE,ALPI
      READ(5,2) RSHC,CDC,RC ,SIC,ALPC,ALPN
      READ(5,2) E,TDC,TDE,T,TR,RADC
    2 FORMAT(6E10.2)
      N=1
      TI=0.
      XMIN=.01
      CMIN=1.E-06
      TMAX=1.E-08
      HCNT=1.
      GE=1./RC+1./RSHE
      GC=1./RC+1./RSHC+1./RB
      X1=.6
      X2=-3.4
      V(1)=X1-X2
      XP(1)=TI
      XS(1,1)=X1
      XS(2,1)=X2
      UTEMP(1,1)=0.
      UTEMP(2,1)=E
    6 IF(TI.GE.1.5E-06) RADC=0.
      T=2.*T
      IF(T.GT.TMAX) T=TMAX
    4 CONE1=SIE*ALPE*EXP(ALPE*X1)
      CONC1=SIC*ALPC*EXP(ALPC*X2)
      CONE2=TDE*CONE1
      CONC2=TDC*CONC1
      CONE3=ALPE*CONE2
      CONC3=ALPC*CONC2
      CONE4=SIE*(EXP(ALPE*X1)-1.)
      CONC4=SIC*(EXP(ALPC*X2)-1.)
      DENE=CDE+CONE2
      DENC=CDC+CONC2
      F(1,1)=(-GE*X1+X2/RC+ALPI*CONC4-CONE4)/DENE
      F(2,1)=(-GC*X2+X1/RC+ALPN*CONE4-CONC4)/DENC
      B(1,1)=1./DENE
      B(1,2)=1./(DENE*RC)
      B(2,1)=0.
      B(2,2)=-1./(DENC*RC)
      U(2,1)=E
      DER(1,1)=-(GE+CONE1+CONE3*F(1,1))/DENE
      DER(1,2)=(1./RC+ALPI*CONC1)/DENE
      DER(2,1)=(1./RC+ALPN*CONE1)/DENC
      DER(2,2)=-(GC+CONC1+CONC3*F(2,1))/DENC
      BDER(1,2)=0.
      BDER(2,1)=0.
    8 U(1,1)=((2.*TR-T)*UTEMP(1,1)+2.*T*RADC)/(2.*TR+T)
      BDER(1,1)=-(CONE3*(U(1,1)+E/RC))/(DENE**2)
      BDER(2,2)=(CONC3*E)/(RC*(DENC**2))
      CALL ADD (BDER,DER,NS,NS,TEMP1)
      Q=T/2.
      CALL CONST (Q,TEMP1,NS,NS,TEMP)
      CALL SUB (AI,TEMP,NS,NS,TEMP1)
      CALL RECIP (.000001,TEMP1,DERINV,KER,NS)
      Q=T
      CALL CONST  (Q,DERINV,NS,NS,TEMP)
      CALL ADD (U,UTEMP,NS,NU,UT)
      Q=.5
      CALL CONST (Q,UT,NS,NU,UT)
```

```
      CALL PROD (B,UT,NS,NS,NU,TEMP2)
      CALL ADD (TEMP2,F,NS,NU,TEMP1)
      CALL PROD (TEMP,TEMP1,NS,NS,NU,TEMP2)
      CALL ADD  (TEMP2,XS,NS,NU,TEMP)
C     PROCEDURE GOOD FOR SMALL
C        VE(N)-VE(N-1)
C        VC(N)-VC(N-1)
C     DEFINE A NORM = SQRT(DELX1**2+DELX2**2)
      DELX1=TEMP(1,1)-XS(1,1)
      DELX2=TEMP(2,1)-XS(2,1)
      XNORM=SQRT(DELX1**2+DELX2**2)
      CNORM=ABS(U(1,1)-UTEMP(1,1))
      IF(CNORM.GT.CMIN) GO TO 5
      IF(XNORM.GT.XMIN) GO TO 5
      GO TO 7
    5 T=T/2.
      GO TO 8
    7 TI=TI+T
      X1=TEMP(1,1)
      X2=TEMP(2,1)
      Q=1.
      CALL CONST (Q,U,NS,NU,UTEMP)
      CALL CONST (Q,TEMP,NS,NU,XS)
      TEMT=HCNT*1.E-08
      IF(TI.LT.TEMT) GO TO 6
      HCNT=HCNT+1.
      N=N+1
      XP(N)=TI
      V(N)=X1-X2
      IF(N.LT.903) GO TO 6
      READ(5,11) ITITLE
   11 FORMAT(6A8)
      READ(5,12) LABEL1
   12 FORMAT(A4)
      CALL DRAW(900,XP,V ,0,0,LABEL1,ITITLE,2.E-06,1.,
     10,0,2,2,5,5,1,L)
      WRITE(6,13) L
   13 FORMAT(//,1X,'L=',I10)
  106 RETURN
      END
```

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The use of the digital computer for the simultaneous generation of circuit responses and their corresponding sensitivity to parameter changes is considered for circuits containing diodes and transistors. Modeling of the diodes and transistors is based upon the Ebers-Moll equations, with the addition of voltage-dependent capacitors to account for switching time. The resulting general iterative equations are then used to calculate the effects of radiation on a p-n junction. The waveforms and circuit recovery time are studied as a function of circuit parameters. The use of sensitivity functions for predicting waveform variations is considered.

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